Calculus: Inclass Homework 11

December 13th, 2007

1. Consider the region in the first quadrant bounded by $x = y - y^3$ and the y-axis. Find the volume of the solid obtained by rotating the region about the y-axis and the line $y = 1$.

2. Let a, b be non-negative integers.

    (a) For $a \geq 1$ show that

    $$\int_0^1 t^a(1-t)^b \mathrm{d}t = \frac{a}{b+1} \int_0^1 t^{a-1}(1-t)^{b+1} \mathrm{d}t.$$

    (b) Deduce from part (a) that

    $$\int_0^1 t^a(1-t)^b \mathrm{d}t = \frac{a!b!}{(a+b+1)!}.$$