

Calculus: Homework 8

November 15th, 2007

1. A pentagon with a perimeter of 30 cm is to be constructed by adjoining an equilateral triangle to a rectangle. Find the dimensions of the rectangle and triangle that will maximize the area of the pentagon.

2. The reaction of the body to a dose of medicine can sometimes be represented by an equation of the form

$$R = M^2 \left(\frac{C}{2} - \frac{M}{3} \right),$$

where C is a positive constant and M is the amount of medicine absorbed in the blood. Find the amount of medicine to which the body is most sensitive, i.e., find the value of M that maximizes dR/dM .

3. Find all antiderivatives of

$$f(x) = \frac{1}{5} - \frac{2}{x^3} + 2x$$

4. Assume that $f(x)$ and $g(x)$ are functions that are twice differentiable on (a, b) . Find all antiderivatives of

$$f(x)g''(x) - f''(x)g(x)$$

on (a, b) .

5. Assume that for a non-negative function f on an interval $[a, b]$ the area of the region below f and above the x -axis is defined by equation 4 in Section 5.1 if the limit exists and is independent of the choice of the sample points; otherwise we say that the area is undefined. Using this notation, is the area below the function

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is irrational;} \\ 0, & \text{if } x \text{ is rational,} \end{cases} \quad x \in [0, 1]$$

and above the x -axis defined?