

# Calculus: Homework 6

October 25th, 2007

1. Show that

$$(\cos x \cdot e^{-x})^{(4)} = -4 \cos x \cdot e^{-x}.$$

2. Let  $f(x)$  be a given function and  $a \in \mathbb{R}$ . Find the polynomial  $p_n(x)$  of degree  $n$  with

$$p_n^{(i)}(a) = f^{(i)}(a) \quad 0 \leq i \leq n.$$

Which meaning does  $p_1(x)$  have?

Hint: Set  $p_n(x) = \sum_{i=0}^n a_i(x-a)^i$ .

3. Find absolute extreme values of

$$f(x) = x^2 - 2 - \cos x$$

on  $[-1, 3]$ .

4. Consider the function  $f(x) = x(x^2 - x - 2)$  on  $[-1, 1]$ . Check whether the assumptions of the mean value theorem are satisfied. If yes, find all  $c \in (-1, 1)$  with

$$f'(c) = \frac{f(1) - f(-1)}{1 - (-1)}.$$

5. Assume that for  $C > 0$  and  $\alpha > 1$ , we have

$$|f(x) - f(y)| \leq C|x - y|^\alpha$$

for all  $x, y \in (a, b)$ . Show that  $f$  must be constant. If  $\alpha = 1$ , is the latter conclusion still true?

Hint: Try  $f(x) = \sin x$ .