

Calculus: Homework 5

October 18th, 2007

1. Let $g = \sqrt{x(x+n)}$ and $a = (x + (x+n))/2$ denote the geometric and arithmetic mean of x and $x+n$, respectively. Show that $dg/dx = a/g$.

2. Find all points where the tangent line to

$$25x^2 + 16y^2 + 200x - 160y + 400 = 0$$

is either horizontal or vertical.

3. Find equations of the four tangent lines to

$$x^4 = 4(4x^2 - y^2)$$

at $y = 3$.

4. Let the following function be given

$$y = \frac{x}{\sqrt{a^2 - 1}} - \frac{2}{\sqrt{a^2 - 1}} \arctan \frac{\sin x}{a + \sqrt{a^2 - 1} + \cos x}.$$

Show that $y' = (a + \cos x)^{-1}$.

5. A swimming pool is 12 m long and 6 m wide. The depth is linearly increasing between 1 m and 3 m from the left to the right endpoint of the pool (along the length). Assume that at the moment the water level at the deepest point is 1 m and that water is pumped into the pool at rate $1/4 \text{ m}^2/\text{min}$. Find the rate at which the water level is currently rising.