

# Calculus: Homework 4

October 11th, 2007

1. Find  $a, b, c$  such that

$$y = ax^2 + bx + c$$

passes through  $(0, 1)$  and has  $y = x - 1$  as tangent at  $(1, 0)$ .

2. Find all common tangents of  $y = x^2$  and  $y = -x^2 + 6x - 5$ .

3. Compute the derivative of

$$f(x) = x \left( 1 - \frac{4}{x+3} \right).$$

4. Show that the following two functions

$$f(x) = \frac{3x}{x+2}, \quad g(x) = \frac{5x+4}{x+2}$$

have identical derivatives. Can you find a relation between  $f(x)$  and  $g(x)$ ?

5. Prove the following differentiation rules.

(a)  $(\sec x)' = \sec x \tan x$ .

(b)  $(\cot x)' = -\csc^2 x$ .

6. Compute the following limit

$$\lim_{x \rightarrow 0} \frac{\sin(2^n x) - 2^n \sin(x)}{x^2},$$

where  $n$  is a non-negative integer.