Calculus: Homework 3

October 4th, 2007

1. Assume that the occupancy of the storage of a company at time t is given by

$$N(t) = 50\left(2\left\lfloor\frac{t+2}{2}\right\rfloor - t\right),$$

where $\lfloor x \rfloor$ denotes the largest integer $\leq x$.

(a) Where is $N(t)$ continuous?

(b) At which time points is the storage refilled?

2. Assume that you swim from one shore of a river to the other, where you start at some point A and finish at some point B. Use the intermediate value theorem to show that for every path, there exist a point where your distance from A is the same as from B. What are the assumptions you are making?

3. Let f be differentiable at a. Prove that

$$\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h} = f'(a).$$

4. Assume that for all x in some open interval containing 1, we have

$$-2(x-1)^2 \leq f(x) \leq 3(x-1)^2.$$

Is the function $f(x)$ differentiable at 1? If yes, what is the value of $f'(1)$? If no, why not?

5. Use the definition of the derivative to find f' of

$$f(x) = \sqrt{1-2x}.$$

Also, state the domain of f and f'.

6. Find the derivative of $f(x) = x|x|$ and state its domain.