

Calculus: Homework 2

September 27th, 2007

1. Use the ϵ - δ definition of the limit to show that

$$\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3} = 6.$$

2. Consider the following function

$$f(x) = \begin{cases} 2x, & \text{if } x \leq 1; \\ 4 - x, & \text{if } x > 1. \end{cases}$$

Use the ϵ - δ definition of the limit to show that $\lim_{x \rightarrow 1} f(x)$ does not exist.

3. Use the limit laws to find the following limit

$$\lim_{x \rightarrow 0} \left(\frac{\sqrt{x^2 + 1} - 1}{x^2} \right)^3.$$

Carefully explain at each step which limit law you have used.

4. Assume that $\lim_{x \rightarrow a} f(x)$ exists and $\lim_{x \rightarrow a} (f(x) + g(x))$ does not exist. Show that then $\lim_{x \rightarrow a} g(x)$ does not exist as well.
5. Find all horizontal asymptotes of

$$f(x) = \frac{3x - 1}{\sqrt[4]{16x^4 - 2x}}.$$

6. Consider the line which passes through the point $(0, 4)$ and has slope m . Let $d(m)$ denote the shortest distance of $(3, 1)$ to the line.
- (i) Find $d(m)$.
- (ii) Compute $\lim_{m \rightarrow \infty} d(m)$ and $\lim_{m \rightarrow -\infty} d(m)$. Interpret your results geometrically.