Calculus: Homework 11

December 6th, 2007

1. Sketch the region enclosed by $y = x^2$ and $y = 2 - |x|$ and use the method of cylindrical shells to find the volume of the solid obtained by rotating it about the x-axis.

2. (a) Let f have a continuous second derivative. Show that

$$f(b) - f(a) = f'(a)(b-a) - \int_a^b f''(x)(x-b)\mathrm{d}x.$$

   (b) Let f have a continuous third derivative. Show that

$$f(b) - f(a) = f'(a)(b-a) + \frac{f''(a)}{2}(b-a)^2 + \int_a^b \frac{f'''(x)}{2}(x-b)^2\mathrm{d}x.$$

   (c) Generalize part (a) and part (b) to f with a continuous n-th order derivative.

3. Evaluate the integrals

$$\int t^n \ln t \mathrm{d}t, \qquad \int t^n (\ln t)^2 \mathrm{d}t,$$

   where $n \geq 0$ is an integer.

4. Evaluate the integral

$$\int (\sin 3x - \sin x)^2 \mathrm{d}x.$$

5. Prove the following recursion formula

$$\int \csc^n x \mathrm{d}x = -\frac{\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} x \mathrm{d}x$$

   for $n \geq 2$ and use it to obtain

$$\int \csc^3 x \mathrm{d}x \quad \text{and} \quad \int \csc^5 x \mathrm{d}x.$$