Calculus: Homework 10

November 29th, 2007

1. Evaluate
$$\int x^2\sqrt{x+1}\mathrm{d}x.$$

2. Evaluate the following integral
$$I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}\mathrm{d}x,$$
where f is a continuous function. Use it to find
$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\mathrm{d}x.$$

3. Let f be a continuous function on $\mathbb{R}$ satisfying
$$f(x+y) = f(x)\cdot f(y), \qquad \forall x, y \in \mathbb{R}.$$
Show that either $f \equiv 0$ or $f(x) = a^x$ for some positive constant a.
Hint: Find $f(x)$ first for $x \in \mathbb{N}$, then $x \in \mathbb{Z}$, then $x \in \mathbb{Q}$, etc.

4. Sketch the region bounded by $y = 6 - x^2$, $y = x$ $(x < 0)$, and $y = -x$ $(x > 0)$ and find its area.

5. Sketch the region bounded by the curves $x + y = 3, 2x + y = 6$, and $x = 0$ and find the volume of the solid obtained by rotating the region about the y-axis.