# Calculus: Homework 1

September 13th, 2007

1. Consider the following two functions on $\mathbb{R}$

$$y = \frac{e^x - e^{-x}}{2} =: \sinh(x), \qquad y = \frac{e^x + e^{-x}}{2} =: \cosh(x)$$

   which are called *hyperbolic sine* and *hyperbolic cosine*.

   (i) Show that for all $x \in \mathbb{R}$,
   $$\cosh^2(x) - \sinh^2(x) = 1.$$

   (ii) Plot the two functions by starting from the graph of $e^x$ and using a similar method as in Example 1 on page 57 in our textbook.

   (iii) Are the functions one-to-one? If no, restrict the domains such that the resulting functions are one-to-one. Then, find formulas for the inverse functions.

2. Evaluate $(\log_2 3)(\log_3 4)(\log_4 5) \cdots (\log_{31} 32)$.

3. Simplify $\cos(2\sin^{-1}(x))$.

4. Find the following limit or argue that it does not exist
$$\lim_{x \to 5} \frac{|x-5|}{x-5}.$$

5. Use your calculator to compute values of $f(x) = (1+x)^{1/x}$ for $x$ close to $0$. Based on your results, what is your guess for $\lim_{x \to 0} f(x)$?

6. Consider the following function

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational;} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

   What is your guess for $\lim_{x \to a} f(x)$, where $a$ is an arbitrary real number?