62. $y=\left(\frac{2x-3}{2x+5}\right)^{2x+1} \Rightarrow \ln y=(2x+1)\ln\left(\frac{2x-3}{2x+5}\right) \Rightarrow$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(2x-3)-\ln(2x+5)}{1/(2x+1)} = \lim_{x\to\infty} \frac{2/(2x-3)-2/(2x+5)}{-2/(2x+1)^2} = \lim_{x\to\infty} \frac{-8(2x+1)^2}{(2x-3)(2x+5)}$$

$$= \lim_{x\to\infty} \frac{-8(2+1/x)^2}{(2-3/x)(2+5/x)} = -8 \Rightarrow \lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1} = e^{-8}$$

77. **(a)** We show that $\lim_{x\to 0} \frac{f(x)}{x^n}=0$ for every integer $n\geq 0$. Let $y=\frac{1}{x^2}$. Then

$$\lim_{x\to 0} \frac{f(x)}{x^{2n}} = \lim_{x\to 0} \frac{e^{-1/x^2}}{\left(x^2\right)^n} = \lim_{y\to\infty} \frac{y^n}{e^y} = \lim_{y\to\infty} \frac{ny^{n-1}}{e^y} = \cdots = \lim_{y\to\infty} \frac{n!}{e^y} = 0 \Rightarrow$$

$\lim_{x\to 0} \frac{f(x)}{x^n} = \lim_{x\to 0} x^n \frac{f(x)}{x^{2n}} = \lim_{x\to 0} x^n \lim_{x\to 0} \frac{f(x)}{x^{2n}} = 0$. Thus, $f'(0)=\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{f(x)}{x} = 0$.

(b) Using the Chain Rule and the Quotient Rule we see that $f^{(n)}(x)$ exists for $x\neq 0$. In fact, we prove by induction that for each $n\geq 0$, there is a polynomial p_n and a non–negative integer k_n with $f^{(n)}(x)=p_n(x)f(x)/x^{k_n}$ for $x\neq 0$. This is true for $n=0$; suppose it is true for the n th derivative. Then $f'(x)=f(x)(2/x^3)$, so

$$\begin{aligned} f^{(n+1)}(x) &= \left[x^{k_n}\left[p_n'(x)f(x)+p_n(x)f'(x)\right]-k_n x^{k_n-1} p_n(x)f(x)\right] x^{-2k_n} \\ &= \left[x^{k_n} p_n'(x)+p_n(x)\left(2/x^3\right)-k_n x^{k_n-1} p_n(x)\right] f(x)x^{-2k_n} \\ &= \left[x^{k_n+3} p_n'(x)+2p_n(x)-k_n x^{k_n+2} p_n(x)\right] f(x)x^{-\left(2k_n+3\right)} \end{aligned}$$

which has the desired form.

Now we show by induction that $f^{(n)}(0)=0$ for all n . By part (a), $f'(0)=0$. Suppose that $f^{(n)}(0)=0$. Then

$$\begin{aligned} f^{(n+1)}(0) &= \lim_{x\to 0} \frac{f^{(n)}(x)-f^{(n)}(0)}{x-0} = \lim_{x\to 0} \frac{f^{(n)}(x)}{x} = \lim_{x\to 0} \frac{p_n(x)f(x)/x^{k_n}}{x} = \lim_{x\to 0} \frac{p_n(x)f(x)}{x^{k_n+1}} \\ &= \lim_{x\to 0} p_n(x) \lim_{x\to 0} \frac{f(x)}{x^{k_n+1}} = p_n(0)\cdot 0 = 0 \end{aligned}$$

52. $y=f(x)=\tan^{-1}\left(\frac{x-1}{x+1}\right)$ **A.** $D=\{x\,|\,x\neq -1\}$

B. x –intercept $=1$, y –intercept

$=f(0)=\tan^{-1}(-1)=-\frac{\pi}{4}$ **C.** No symmetry **D.**

$\lim_{x\to\pm\infty}\tan^{-1}\left(\frac{x-1}{x+1}\right)=\lim_{x\to\pm\infty}\tan^{-1}\left(\frac{1-1/x}{1+1/x}\right)=\tan^{-1}1=\frac{\pi}{4}$, so $y=\frac{\pi}{4}$ is a HA. Also

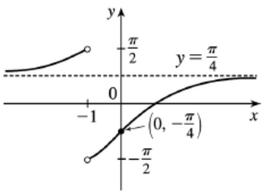
$\lim_{x\to-1^+}\tan^{-1}\left(\frac{x-1}{x+1}\right)=-\frac{\pi}{2}$ and $\lim_{x\to-1^-}\tan^{-1}\left(\frac{x-1}{x+1}\right)=\frac{\pi}{2}$. **E.**

$$f'(x)=\frac{1}{1+[(x-1)/(x+1)]^2}\,\frac{(x+1)-(x-1)}{(x+1)^2}$$

$$=\frac{2}{(x+1)^2+(x-1)^2}=\frac{1}{x^2+1}>0$$

so f is increasing on $(-\infty,-1)$ and $(-1,\infty)$. **F.** No extreme values

H.



G. $f''(x)=-2x/\left(x^2+1\right)^2>0\Leftrightarrow x<0$, so f is CU on $(-\infty,-1)$ and $(-1,0)$, and CD on $(0,\infty)$.
IP at $\left(0,-\frac{\pi}{4}\right)$