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43. **(a)** For a sphere of radius r, the circumference is $C=2\pi r$ and the surface area is $S=4\pi r^2$, so $r=C/(2\pi)\Rightarrow S=4\pi(C/2\pi)^2=C^2/\pi\Rightarrow dS=(2/\pi)C\,dC$. When $C=84$ and $dC=0.5$, $dS=\frac{2}{\pi}(84)(0.5)=\frac{84}{\pi}$, so the maximum error is about $\frac{84}{\pi}\approx 27\text{ cm}^2$. Relative error $\approx\frac{dS}{S}=\frac{84/\pi}{84^2/\pi}=\frac{1}{84}\approx 0.012$

(b) $V=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{C}{2\pi}\right)^3=\frac{C^3}{6\pi^2}\Rightarrow dV=\frac{1}{2\pi^2}C^2\,dC$. When $C=84$ and $dC=0.5$,

$dV=\frac{1}{2\pi^2}(84)^2(0.5)=\frac{1764}{\pi^2}$, so the maximum error is about $\frac{1764}{\pi^2}\approx 179\text{ cm}^3$. The relative error is approximately $\frac{dV}{V}=\frac{1764/\pi^2}{(84)^3/(6\pi^2)}=\frac{1}{56}\approx 0.018$.

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37. $g(x)=|2x+3|=\begin{cases}2x+3 & \text{if } 2x+3\geq 0\\ -(2x+3) & \text{if } 2x+3<0\end{cases}\Rightarrow g'(x)=\begin{cases}2 & \text{if } x>-\frac{3}{2}\\ -2 & \text{if } x<-\frac{3}{2}\end{cases}$ $g'(x)$ is never 0, but

$'$

$x=-\frac{3}{2}$, so $-\frac{3}{2}$ is the only critical number.

38. $g(x)=x^{1/3}-x^{-2/3}\Rightarrow g'(x)=\frac{1}{3}x^{-2/3}+\frac{2}{3}x^{-5/3}=\frac{1}{3}x^{-5/3}(x+2)=\frac{x+2}{3x^{5/3}}$.

$g'(-2)=0$ and $g'(0)$ does not exist, but 0 is not in the domain of g, so the only critical number is -2.

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20. $f(x)=x^4+4x+c$. Suppose that $f(x)=0$ has three distinct real roots a, b, d where $a<b<d$. Then $f(a)=f(b)=f(d)=0$. By Rolle's Theorem there are numbers c_1 and c_2 with $a<c_1<b$ and $b<c_2<d$ and $0=f'\left(c_1\right)=f'\left(c_2\right)$, so $f'(x)=0$ must have at least two real solutions. However $0=f'(x)=4x^3+4=4\left(x^3+1\right)=4(x+1)\left(x^2-x+1\right)$ has as its only real solution $x=-1$. Thus, $f(x)$ can have at most two real roots.