Stewart Calculus ET 5e 0534393217;3. Differentiation Rules; 3.5 The Chain Rule

40. $y=\sqrt{x+\sqrt{x+\sqrt{x}}} \Rightarrow y'=\frac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-1/2}\left[1+\frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right]$

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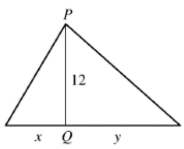
37. $y=\dfrac{\sin^2 x\tan^4 x}{(x^2+1)^2} \Rightarrow \ln y=\ln(\sin^2 x\tan^4 x)-\ln(x^2+1)^2 \Rightarrow$

$\ln y=\ln(\sin x)^2+\ln(\tan x)^4-\ln(x^2+1)^2 \Rightarrow \ln y=2\ln|\sin x|+4\ln|\tan x|-2\ln(x^2+1)\Rightarrow$

$\frac{1}{y}y'=2\cdot\frac{1}{\sin x}\cdot\cos x+4\cdot\frac{1}{\tan x}\cdot\sec^2 x-2\cdot\frac{1}{x^2+1}\cdot 2x\Rightarrow$

$$y'=\frac{\sin^2 x\tan^4 x}{(x^2+1)^2}\left(2\cot x+\frac{4\sec^2 x}{\tan x}-\frac{4x}{x^2+1}\right)$$

Stewart Calculus ET 5e 0534393217;3. Differentiation Rules; 3.10 Related Rates



32.

Using Q for the origin, we are given $\frac{dx}{dt}=-2$ ft / s and need to find $\frac{dy}{dt}$ when $x=-5$. Using the Pythagorean Theorem twice, we have $\sqrt{x^2+12^2}+\sqrt{y^2+12^2}=39$, the total length of the rope. Differentiating with respect to t , we get $\frac{x}{\sqrt{x^2+12^2}}\frac{dx}{dt}+\frac{y}{\sqrt{y^2+12^2}}\frac{dy}{dt}=0$, so

$\frac{dy}{dt}=-\frac{x\sqrt{y^2+12^2}}{y\sqrt{x^2+12^2}}\frac{dx}{dt}$. Now when $x=-5$, $39=\sqrt{(-5)^2+12^2}+\sqrt{y^2+12^2}=13+\sqrt{y^2+12^2} \Leftrightarrow \sqrt{y^2+12^2}=26$

, and $y=\sqrt{26^2-12^2}=\sqrt{532}$. So when $x=-5$, $\frac{dy}{dt}=-\frac{(-5)(26)}{\sqrt{532}(13)}(-2)=-\frac{10}{\sqrt{133}}\approx-0.87$ ft / s. So cart B is moving towards Q at about 0.87 ft / s.