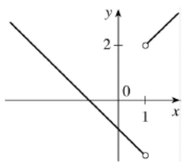
47. **(a)**

(i) $\lim_{x\to 1^+} \frac{x^2-1}{|x-1|} = \lim_{x\to 1^+} \frac{x^2-1}{x-1} = \lim_{x\to 1^+} (x+1) = 2$

(ii) $\lim_{x\to 1^-} \frac{x^2-1}{|x-1|} = \lim_{x\to 1^-} \frac{x^2-1}{-(x-1)} = \lim_{x\to 1^-} -(x+1) = -2$

(b) No, $\lim_{x\to 1} F(x)$ does not exist since $\lim_{x\to 1^+} F(x) \neq \lim_{x\to 1^-} F(x)$.

(c)



13. **(a)** $A=\pi r^2$ and $A=1000\text{ cm}^2 \Rightarrow \pi r^2=1000 \Rightarrow r^2=\frac{1000}{\pi} \Rightarrow$

$r=\sqrt{\frac{1000}{\pi}}\ [r>0] \approx 17.8412$ cm.

(b) $|A-1000| \leq 5 \Rightarrow -5 \leq \pi r^2 - 1000 \leq 5 \Rightarrow 1000-5 \leq \pi r^2 \leq 1000+5 \Rightarrow$
$\sqrt{\frac{995}{\pi}} \leq r \leq \sqrt{\frac{1005}{\pi}} \Rightarrow 17.7966 \leq r \leq 17.8858$. $\sqrt{\frac{1000}{\pi}} - \sqrt{\frac{995}{\pi}} \approx 0.04466$ and
$\sqrt{\frac{1005}{\pi}} - \sqrt{\frac{1000}{\pi}} \approx 0.04455$. So if the machinist gets the radius within 0.0445 cm of 17.8412 ,
the area will be within 5 cm^2 of 1000 .

(c) x is the radius, $f(x)$ is the area, a is the target radius given in part (a), L is the target area (1000) , ε is the tolerance in the area (5), and δ is the tolerance in the radius given in part (b).

42. Given $M>0$, we need $\delta>0$ such that $0<|x+3|<\delta \Rightarrow 1/(x+3)^4>M$. Now $\frac{1}{(x+3)^4}>M \Leftrightarrow (x+3)^4<\frac{1}{M} \Leftrightarrow$
$|x+3|<\frac{1}{\sqrt[4]{M}}$. So take $\delta=\frac{1}{\sqrt[4]{M}}$. Then $0<|x+3|<\delta=\frac{1}{\sqrt[4]{M}} \Rightarrow \frac{1}{(x+3)^4}>M$, so $\lim_{x\to -3} \frac{1}{(x+3)^4}=\infty$.

53.

$\lim_{x\to\infty} \frac{4x-1}{x} = \lim_{x\to\infty} \left(4-\frac{1}{x}\right) = 4$, and $\lim_{x\to\infty} \frac{4x^2+3x}{x^2} = \lim_{x\to\infty} \left(4+\frac{3}{x}\right) = 4$. Therefore, by the Squeeze Theorem, $\lim_{x\to\infty} f(x)=4$.