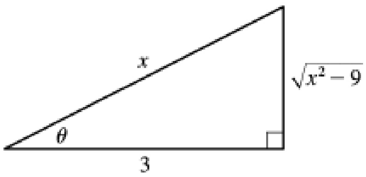
*Stewart Calculus ET 5e 0534393217;7. Techniques of Integration; 7.3 Trigonometric Substitution*

13. Let $x=3\sec\theta$ , where $0\leq\theta<\frac{\pi}{2}$ or $\pi\leq\theta<\frac{3\pi}{2}$ . Then $dx=3\sec\theta\ \tan\theta\ d\theta$ and $\sqrt{x^2-9}=3\tan\theta$ so

$$\int\frac{\sqrt{x^2-9}}{x^3}\,dx=\int\frac{3\tan\theta}{27\sec^3\theta}\,3\sec\theta\ \tan\theta\ d\theta=\frac{1}{3}\int\frac{\tan^2\theta}{\sec^2\theta}\,d\theta$$




$$=\frac{1}{3}\int\sin^2\theta\ d\theta=\frac{1}{3}\int\frac{1}{2}(1-\cos 2\theta\ )d\theta=\frac{1}{6}\theta-\frac{1}{12}\sin 2\theta\ +C=\frac{1}{6}\theta-\frac{1}{6}\sin\theta\ \cos\theta\ +C$$

$$=\frac{1}{6}\sec^{-1}\left(\frac{x}{3}\right)-\frac{1}{6}\,\frac{\sqrt{x^2-9}}{x}\,\frac{3}{x}+C=\frac{1}{6}\sec^{-1}\left(\frac{x}{3}\right)-\frac{\sqrt{x^2-9}}{2x^2}+C$$

*Stewart Calculus ET 5e 0534393217;7. Techniques of Integration; 7.4 Integration of Rational Functions by Partial Fractions*

55. **(a)** If $t=\tan\left(\frac{x}{2}\right)$ , then $\frac{x}{2}=\tan^{-1}t$ . The figure gives $\cos\left(\frac{x}{2}\right)=\frac{1}{\sqrt{1+t^2}}$ and $\sin\left(\frac{x}{2}\right)=\frac{t}{\sqrt{1+t^2}}$ .

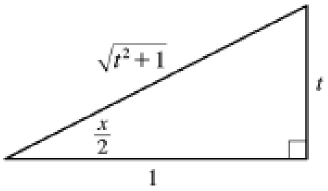
**(b)** $\cos x=\cos\left(2\cdot\frac{x}{2}\right)=2\cos^2\left(\frac{x}{2}\right)-1$

$=2\left(\frac{1}{\sqrt{1+t^2}}\right)^2-1=\frac{2}{1+t^2}-1=\frac{1-t^2}{1+t^2}$

$\sin x=\sin\left(2\cdot\frac{x}{2}\right)=2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)=2\,\frac{t}{\sqrt{1+t^2}}\,\frac{1}{\sqrt{1+t^2}}=\frac{2t}{1+t^2}$

**(c)**

$$\frac{x}{2}=\arctan t \Rightarrow x=2\arctan t \Rightarrow dx=\frac{2}{1+t^2}\,dt$$




56. Let $t=\tan(x/2)$ . Then, using Exercise 55 , $dx=\frac{2}{1+t^2}\,dt$ , $\sin x=\frac{2t}{1+t^2} \Rightarrow$

$$\int \frac{dx}{3-5\sin x} = \int \frac{2\,dt/\left(1+t^2\right)}{3-10t/\left(1+t^2\right)} = \int \frac{2\,dt}{3\left(1+t^2\right)-10t} = 2\int \frac{dt}{3t^2-10t+3}$$

$$= \frac{1}{4}\int \left[ \frac{1}{t-3} - \frac{3}{3t-1} \right] dt = \frac{1}{4}\left(\ln|t-3| - \ln|3t-1|\right)+C = \frac{1}{4}\ln \left| \frac{\tan(x/2)-3}{3\tan(x/2)-1} \right| + C$$