

38. Use shells:

$$\begin{aligned} V &= \int_1^2 2\pi x (-x^2 + 3x - 2) dx = 2\pi \int_1^2 (-x^3 + 3x^2 - 2x) dx \\ &= 2\pi \left[-\frac{1}{4}x^4 + x^3 - x^2 \right]_1^2 = 2\pi \left[(-4 + 8 - 4) - \left(-\frac{1}{4} + 1 - 1 \right) \right] = \frac{\pi}{2} \end{aligned}$$

33. Let $w = \sqrt{x}$, so that $x = w^2$ and $dx = 2w dw$. Thus, $\int \sin \sqrt{x} dx = \int 2w \sin w dw$. Now use parts with $u = 2w$, $dv = \sin w dw$, $du = 2 dw$, $v = -\cos w$ to get

$$\begin{aligned} \int 2w \sin w dw &= -2w \cos w + \int 2 \cos w dw = -2w \cos w + 2 \sin w + C \\ &= -2\sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x} + C = 2(\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}) + C \end{aligned}$$

66. $\int_{-\pi}^{\pi} \sin mx \sin nx dx = \int_{-\pi}^{\pi} \frac{1}{2} [\cos (m-n)x - \cos (m+n)x] dx$. If $m \neq n$, this is equal to

$$\begin{aligned} \frac{1}{2} \left[\frac{\sin (m-n)x}{m-n} - \frac{\sin (m+n)x}{m+n} \right]_{-\pi}^{\pi} &= 0. \text{ If } m=n, \text{ we get} \\ \int_{-\pi}^{\pi} \frac{1}{2} [1 - \cos (m+n)x] dx &= \left[\frac{1}{2}x \right]_{-\pi}^{\pi} - \left[\frac{\sin (m+n)x}{2(m+n)} \right]_{-\pi}^{\pi} = \pi - 0 = \pi. \end{aligned}$$