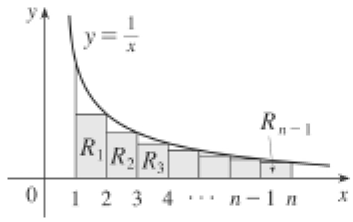
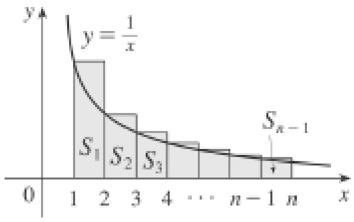
Stewart Calculus ET 5e 0534393217;5. Integrals; 5.6 The Logarithm Defined as an Integral

3.



The area of R_i is $\frac{1}{i+1}$ and so $\frac{1}{2}+\frac{1}{3}+\cdot\cdot\cdot+\frac{1}{n}<\int_1^n \frac{1}{t}\ dt=\ln n$.

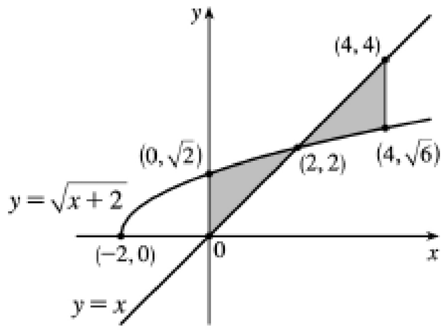


The area of S_i is $\frac{1}{i}$ and so $1+\frac{1}{2}+\cdot\cdot\cdot+\frac{1}{n-1}>\int_1^n \frac{1}{t}\ dt=\ln n$.

Thus, $\frac{1}{2}+\frac{1}{3}+\cdot\cdot\cdot+\frac{1}{n}<\ln n<1+\frac{1}{2}+\cdot\cdot\cdot+\frac{1}{n-1}$.

Stewart Calculus ET 5e 0534393217;6. Application of Integration; 6.1 Areas between Curves

30. The curves intersect when $\sqrt{x+2}=x\Rightarrow x+2=x^2\Rightarrow x^2-x-2=0\Rightarrow$ $(x-2)(x+1)=0\Rightarrow x=-1$ or 2 .



$$A=\int_0^4 \left|\sqrt{x+2}-x\right|\ dx$$

$$=\int_0^2 \left(\sqrt{x+2}-x\right)dx+\int_2^4 \left(x-\sqrt{x+2}\ \right)dx$$

$$=\left[\ \frac{2}{3}(x+2)^{3/2}-\frac{1}{2}x^2\right]_0^2+\left[\ \frac{1}{2}x^2-\frac{2}{3}(x+2)^{3/2}\right]_2$$

$$=\left(\ \frac{16}{3}-2\right)-\left(\ \frac{2}{3}\ \left(2\sqrt{2}\ \right)-0\right)+\left(8-\frac{2}{3}\ \left(6\sqrt{6}\ \right)\right)-\left(2-\frac{16}{3}\ \right)$$

$$=4+\frac{32}{3}-\frac{4}{3}\sqrt{2}-4\sqrt{6}=\frac{44}{3}-4\sqrt{6}-\frac{4}{3}\sqrt{2}$$

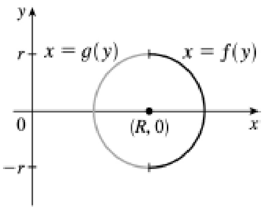
Stewart Calculus ET 5e 0534393217;6. Application of Integration; 6.2 Volumes

61. **(a)** The torus is obtained by rotating the circle $(x-R)^2+y^2=r^2$ about the $y-$ axis. Solving for x , we see that the right half of the circle is given by $x=R+\sqrt{r^2-y^2}=f(y)$ and the left half by $x=R-\sqrt{r^2-y^2}=g(y)$. So

$$V=\pi\int_{-r}^{r}\left\{[f(y)]^2-[g(y)]^2\right\}\,dy$$

$$=2\pi\int_0^r\left[\left(R^2+2R\sqrt{r^2-y^2}+r^2-y^2\right)-\left(R^2-2R\sqrt{r^2-y^2}+r^2-y^2\right)\right]dy$$

$$=2\pi\int_0^r 4R\sqrt{r^2-y^2}\,dy=8\pi R\int_0^r\sqrt{r^2-y^2}\,dy$$



(b) Observe that the integral represents a quarter of the area of a circle with radius r , so

$$8\pi R\int_0^r\sqrt{r^2-y^2}\,dy=8\pi R\cdot\frac{1}{4}\pi r^2=2\pi^2r^2R\ .$$