Calculus II 0314 Midterm.

1. (a) (3%) Find the scalar and vector projection of $b =< 1, 1, 1 >$ onto $a =< 4, 2, 0 >$. $\frac{3}{\sqrt{5}}$; $\frac{3}{5} < 2, 1, 0 >$

   (b) (3%) Find the plane passes through the point $(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$. $x - 2y + 4z = -1$

   (c) (3%) Find the equation of the tangent line to the curve $x = t^5$, $y = t^4$, $z = t^3$ at the point $(1, 1, 1)$. $\frac{x-1}{5} = \frac{y-1}{4} = \frac{z-1}{3}$

2. (6%) Match the function with its graph (labeled I-VI). Give reasons for your choices.

   (a) $f(x, y) = |x| + |y|$. VI

   (b) $f(x, y) = |xy|$. V

   (c) $f(x, y) = \frac{1}{1+x^2+y^2}$. I

   (d) $f(x, y) = (x^2 - y^2)^2$. IV

   (e) $f(x, y) = (x - y)^2$. II

   (f) $f(x, y) = \sin(|x| + |y|)$. III

3. (a) (3%) Let $f(x, y) = (x^2 + y^2)^{-\frac{5}{2}} e^{\sin(x^2y^3)}$. Find $f_x(1, 0)$. $-5$

   (b) (6%) Let $u(x, t) = f(x + at) + g(x - at)$, where $f$ and $g$ are twice differentiable functions of a single variable. Find $u_{tt}$ and $u_{xx}$. $a^2[f''(x + at) + g''(x - at)]$; $f''(x + at) + g''(x - at)$

4. Justify your answers to get full credit.

   (a) (3%) Does the convergence of $\sum_{n=1}^{\infty} a_n$ imply the convergence of $\sum_{n=1}^{\infty}(a_n + a_{n+1})$? Y

   (b) (3%) Does the convergence of $\sum_{n=1}^{\infty}(a_n + a_{n+1})$ imply the convergence of $\sum_{n=1}^{\infty} a_n$? N

   (c) (3%) Does the convergence of $\sum_{n=1}^{\infty}(|a_n| + |a_{n+1}|)$ imply the convergence of $\sum_{n=1}^{\infty} |a_n|$? Y

5. Let $f(x, y) = \begin{cases} \frac{2x^3y - xy^3}{x^2+y^2}, & \text{if}(x, y) \neq (0, 0) \\ 0, & \text{if}(x, y) = (0, 0) \end{cases}$.

   (a) (3%) Find $f_x(x, y)$ and $f_y(x, y)$ when $(x, y) \neq (0, 0)$. $f_x = \frac{2x^4y+7x^2y^3-y^5}{(x^2+y^2)^2}$; $f_y = \frac{2x^5-5x^3y^2-xy^4}{(x^2+y^2)^2}$.

   (b) (3%) Find $f_x(0, 0)$ and $f_y(0, 0)$. 0; 0

   (c) (3%) Is $f_x(x, y)$ continuous at $(0, 0)$? Y

(d) (3%) Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$. $-1$; $2$

6. Find the limit, if it exists, or show that the limit does not exist.

(a) (3%) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$. N

(b) (3%) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$. N

(c) (3%) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$. 0

(d) (3%) For what values of the number $r$ is the function

$$f(x,y,z) = \begin{cases} \frac{(x+y+z)^r}{x^2+y^2+z^2}, & \text{if}(x,y,z) \neq (0,0,0) \\ 0, & \text{if}(x,y,z) = (0,0,0) \end{cases}$$

continuous on $\mathbb{R}$? $r > 2$

7. (a) (3%) Find the power series expansion of $\ln(1+x)$ and $\ln(1-x)$, where $|x| < 1$. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$; $\displaystyle\sum_{n=1}^{\infty}\frac{-x^n}{n}$

(b) (3%) Using (a) to find the power series expansion of $\ln(\frac{1+x}{1-x})$, where $|x| < 1$. $\displaystyle\sum_{n=0}^{\infty}\frac{2}{2n+1}x^{2n+1}$

(c) (3%) Let $f(x) = x(1-x)^{\frac{3}{2}}$. Find $f^{(6)}(0)$. $\frac{135}{16}$

(d) (3%) Find the power series expansion of $\sin^{-1} x$.(Hint:$\frac{d\sin^{-1}x}{dx} = \frac{1}{\sqrt{1-x^2}}$ and binomial series.) $\displaystyle x + \sum_{n=1}^{\infty}\frac{1\cdot 3\cdot \cdots \cdot (2n-1)x^{2n+1}}{2^n n!(2n+1)}$

8. (a) (3%) Find the interval of convergence of the power series $\displaystyle\sum_{k=0}^{\infty}\frac{(2x+1)^k}{3^k}$. $(-2,1)$

(b) (3%) Determine the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}(a^n+b^n)x^n$, where $a,b \geq 0$. $\min\{\frac{1}{a},\frac{1}{b}\}$

9. Determine the series is convergent or divergent. Explain briefly why.

(a) (3%) $1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{n^3} + \cdots$. C

(b) (3%) $\frac{1}{4} + \frac{1}{7} + \frac{1}{10} + \cdots + \frac{1}{3n+1} + \cdots$. D

(c) (3%) $\displaystyle\sum_{m=2}^{\infty}\frac{1}{m\ln m}$. D

(d) (3%) $\displaystyle\sum_{m=0}^{\infty}\frac{m+1}{(m+2)2^m}$. C

(e) (3%) $\sum_{k=1}^{\infty} \frac{k^2}{k^3 + 1000}$. D

(f) (3%) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 2}$. C

10. Determine the series is divergent, conditional convergent or absolutely convergent. Explain briefly why.

(a) (3%) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!}$. A.C.

(b) (3%) $\sum_{n=1}^{\infty} (-1)^n \tan \frac{1}{n}$. C.C

(c) (3%) $\sum_{n=1}^{\infty} \frac{(-1)^n n^{2007}}{(n+2)!}$. A.C.