Calculus 0314

Midterm.

1. (a) State the definition of the derivative of a function f. (5%)

   (b) Use the definition of derivative to find the derivative of $g(x) = \sqrt{1+2x}$. (5%)

   $g'(x) = \frac{1}{\sqrt{1+2x}}$

2. (a) State the definition of a definite integral. (5%)

   (b) Use such definition to evaluate $\int_0^2 (2 - x^2)dx$. (5%)

   $\frac{4}{3}$

3. Find $\frac{dy}{dx}$. (22%)

   (a) $y = \sqrt{x}(x-1)$.

   $\frac{3x-1}{2\sqrt{x}}$

   (b) $y = e^x \cos x$.

   $e^x(\cos x - \sin x)$

   (c) $y = \cot^2(\sin x)$.

   $-2\cot(\sin x) \cdot \csc^2(\sin x) \cdot \cos x$

   (d) $x^2y + xy^2 = 3x$.

   $\frac{3-y^2-2xy}{x^2+2xy}$

   (e) $y = \tan^{-1}\sqrt{x}$.

   $\frac{1}{2\sqrt{x}\sec^2 y} = \frac{1}{2\sqrt{x}(x+1)}$

   (f) $y = (\ln x)^x$.

   $(\ln x)^x(\ln(\ln x) + \frac{1}{\ln x})$

   (g) $y = \int_2^x (t^2 \sin t)dt$.

   $x^2 \sin x$

   (h) $y = \int_{1-3x}^1 \frac{u^3}{1+u^2}du$.

   $\frac{3(1-3x)^3}{1+(1-3x)^2}$

   (i) $y = \int_{\sqrt{x}}^x \frac{e^t}{t}dt$.

   $\frac{2e^x - e^{\sqrt{x}}}{2x}$

(j) $y = \int_0^x x^2 \sin(t^2) dt$.

$2x \int_0^x \sin(t^2) dt + x^2 \sin(x^2)$

(k) $y = \int_0^x (\int_1^{\sin t} \sqrt{1+u^2} du) dt$. Find $\frac{d^2y}{dx^2}$.

$\sqrt{1+\sin^2 x} \cos x$

4. Evaluate the limits. (22%)

(a) $\lim_{t \to 0} (\frac{1}{t\sqrt{1+t}} - \frac{1}{t})$.

$-\frac{1}{2}$

(b) $\lim_{x \to 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1}$.

$\frac{1}{2}$

(c) $\lim_{x \to \infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4}$.

$\frac{1}{2}$

(d) $\lim_{x \to -\infty} (x + \sqrt{x^2 + 2x})$.

-1

(e) $\lim_{h \to 0} \frac{(1+h)^{10} - 1}{h}$.

10

(f) $\lim_{x \to 0} \frac{1 - \cos x}{x^2 + x}$.

0

(g) $\lim_{x \to (\frac{\pi}{2})^-} (\tan x)^{\cos x}$.

1

(h) $\lim_{n \to \infty} \frac{1}{n} (\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}})$.

$\frac{2}{3}$

(i) $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$.

$\frac{1}{4}$

(j) $\lim_{n \to \infty} (\frac{1}{\sqrt{n(n+1)}} + \frac{1}{\sqrt{n(n+2)}} + \cdots + \frac{1}{\sqrt{n(n+n)}})$.

$2(\sqrt{2} - 1)$

(k) $\lim_{x \to 0} \frac{1}{x} \int_0^x (1 - \tan 2t)^{\frac{1}{t}} dt$.

e^{-2}

5. Evaluate the integral or find the general indefinite integral. (14%)

(a) $\int_0^2 x(2+x^5)dx$.

$\frac{156}{7}$

(b) $\int_1^e \frac{x^2+x+1}{x}dx$.

$\frac{1}{2}e^2+e-\frac{1}{2}$

(c) $\int_{-1}^2 (x-2|x|)dx$.

$-\frac{7}{2}$

(d) $\int x^2\sqrt{x^3+1}dx$.

$\frac{2}{9}(x^3+1)^{\frac{3}{2}}+C$

(e) $\int \sqrt{4-t}dt$.

$-\frac{2}{3}(4-t)^{\frac{3}{2}}+C$

(f) $\int \frac{\sin x}{1+\cos^2 x}dx$.

$-\tan^{-1}(\cos x)+C$

(g) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$.

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6. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 P.M.? (4%)

$\frac{720}{13}$ km/h

7. Find the linearization of $f(x)=\cos x$ at $a=\frac{\pi}{2}$. (4%)

$L(f(x))=-x+\frac{\pi}{2}$

8. Let $f(x)=x^{\frac{1}{3}}(x+4)$. (4%)

(a) Find the intervals of increase or decrease.

increase:$(-1,\infty)$; decrease:$(-\infty,-1)$

(b) Find the local maximum and minimum values.

minimum=-3; no local maximum

(c) Find the intervals of concavity and the inflection points.

concave up:$(-\infty,0),(2,\infty)$; concave down:$(0,2)$; inflection point:$(0,0),(2,6\sqrt[3]{2})$

9. The top and bottom margins of a poster are each $6cm$ and the side margins are each $4cm$. If the area of printed material on the poster is fixed at $384cm^2$, find the dimensions of the poster with the smallest area. (4%)

   24×36

10. An observer stands at a point P, one unit away form a track. Two runners start at the point S in the figure and run along the track. One runner runs three times as fast as the other. Find the maximum value of the observer's angle of sight θ between the runners. [Hint: Maximize $\tan\theta$.] (4%)

    $\frac{\pi}{6}$

11. Find the interval $[a, b]$ for which the value of the integral $\displaystyle\int_a^b (2 + x - x^2)dx$ is a maximum. [Hint: Graph $f(x) = 2 + x - x^2$.] (4%)

    $a = -1$, $b = 2$