Calculus **B** 0314

Quiz 6.

(1) Evaluate the integral (10%)

(i) $\int \dfrac{x-9}{(x+5)(x-2)}\,dx.$

(ii) $\int \dfrac{x+4}{x^2+2x+5}\,dx.$

(i) $\ln \dfrac{(x+5)^2}{|x-2|}+C.$

(ii) $\frac{1}{2}\ln(x^2+2x+5)+\frac{3}{2}\tan^{-1}\frac{x+1}{2}+C.$

(2) Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) the Simpson's Rule to approximate the integral $\int_0^2 (1+x^2)\,dx$ with $n=4$. (10%) (a) $\frac{19}{4}$, (b) $\frac{37}{8}$, (c) $\frac{14}{3}$.

(3) Determine whether each integral is convergent or divergent, and evaluate those that are convergent (10%)

(i) $\int_1^\infty \dfrac{1}{(3x+1)^2}\,dx$ $\frac{1}{12}$.

(ii) $\int_1^\infty \dfrac{\ln x}{x}\,dx.$ divergent.

(4) Find the values of p for which the integral converges and evaluate the integral for these values of p. (10%)

(i) $\int_e^\infty \dfrac{1}{x(\ln x)^p}\,dx$

(ii) $\int_0^1 x^p \ln x\,dx$

(i) the integral converges as $p>1$, and equals to $\dfrac{1}{p-1}$.

(ii) the integral converges as $p>-1$, and equals to $\dfrac{-1}{(p+1)^2}$.

(5) Find the arc length of $y=1+6x^{\frac{3}{2}}$, $0\le x\le 1$. (10%) $\dfrac{2}{243}(82\sqrt{82}-1)$.

(6) Find the length of the curve $y=\int_1^x \sqrt{t^3-1}\,dt$, $1\le x\le 4$. (10%) $\dfrac{62}{5}$.

(7) Find the area of the surface obtained by rotating the curve $y=x^3$, $0\le x\le 2$, about the x-axis. (10%) $\dfrac{145\sqrt{145}-1}{27}\pi$.

(8) Find the area of the surface obtained by rotating the circle $x^2+y^2=r^2$ about the line $y=r$. (10%) $4\pi^2 r^2$.

(9) An aquarium 5 ft long, 2 ft wide, and 3 ft deep is full of water. Find (a) the hydrostatic pressure on the bottom of the aquarium, (b) the hydrostatic force on the bottom, and (c) the hydrostatic force on one end of the aquarium. (10%) (a) 187.5 lb/ft^2 (b) 1875 lb (c) 562.5 lb.

(10) Find the centroid of the region bounded by the given curves. (10%)

$y=\sqrt{x}$, $y=x$. $(\dfrac{2}{5},\dfrac{1}{2})$.