Calculus **B** 0314

Quiz 5.

(1) Find the area of the region bounded by

(a) $y = x$, $y = x^2$. (6%) $\frac{1}{6}$.

(b) $y = |x|$, $y = x^2 - 2$. (6%) $\frac{20}{3}$.

(2) Find the volume of the solid obtained by rotating the region by the given curves about the specified line.

(a) $y = x^2$, $y^2 = x$; about the x-axis. (6%) $\frac{3}{10}\pi$.

(b) $y = x$, $y = \sqrt{x}$; about $y = 1$. (6%) $\frac{1}{6}\pi$.

(c) $y = x^2$, $y = 0$, $x = 1$, $x = 2$; about $x = 4$. (6%) $\frac{67}{6}\pi$.

(3) Find the volume of a pyramid with height h and rectangular base with dimensions b and $2b$. (10%) $\frac{2}{3}b^2h$.

(4) Find the volume of the described solid S in the FIGURE 1. The base of S is an elliptical region with boundary curve $9x^2 + 4y^2 = 36$. Cross-sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base. (10%) 24.

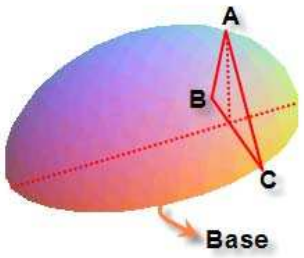

FIGURE 1. $\angle A = 90°$, $\overline{AB} = \overline{AC}$, Base: $9x^2 + 4y^2 = 36$.

(5) Evaluate the integral. (50%)

(i) $\int \ln(2x+1)\,dx$

(ii) $\int \sin^{-1} x\,dx$

(iii) $\int \cos(\ln x)\,dx$

(iv) $\int \sin^3 x \cos^2 x\,dx$

(v) $\int \sec^2 x \tan x\,dx$

(vi) $\int \tan^3 x \sec x\,dx$

(vii) $\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt$

(viii) $\int \frac{dx}{\sqrt{x^2+16}}$

(ix) $\int_0^{\frac{3}{2}} x^3\sqrt{4-9x^2}\,dx$

(x) $\int \frac{dx}{(x^2+2x+2)^2}$

(i) $\frac{1}{2}(2x+1)\ln(2x+1) - x + C$. (ii) $x\sin^{-1} x + \sqrt{1-x^2} + C$. (iii) $\frac{x}{2}(\cos(\ln x) + \sin(\ln x)) + C$.
(iv) $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$. (v) $\frac{1}{2}\sec^2 x + C$. (vi) $\frac{1}{3}\sec^3 x - \sec x + C$.

(vii) $-\frac{1}{4}+\frac{\sqrt{3}}{8}+\frac{\pi}{24}$. (viii) $\ln(\sqrt{1+\frac{x^2}{16}}+\frac{x}{4})+C$.

(ix) (x) $\frac{x+1}{2(x^2+2x+2)}+\frac{1}{2}\tan^{-1}(x+1)+C$.