Calculus **B** 0314

Quiz 4.

(1) Find the absolute maximum and the absolute minimum values of $f(x) = xe^{-x}$ on $[0, 2]$. (10%)

Absolute maximum value is e^{-1}. Absolute minimum value is 0.

(2) Find the critical numbers of $x^{\frac{4}{5}}(x-4)^2$. (10%) The critical numbers are 0, $\frac{8}{7}$ and 4.

(3) Find all numbers c that satisfy the conclusion of the Mean Value Theorem. (10%)

$$f(x) = 3x^2 + 2x + 5, \text{ and } [a, b] = [-1, 1]. \quad c = 0.$$

(4) If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be? (10%) 16.

(5) Find the local extrema and inflection points of $y = \dfrac{x^2}{x^2 + 3}$. (10%) Local minimum point at $x = 0$, and the inflection points at $x = \pm 1$.

(6) Find the limit. (15%)

$(a) \displaystyle\lim_{x \to \infty} \frac{\ln x}{x}$
    $(b) \displaystyle\lim_{x \to 0} \frac{\sin^{-1} x}{x}$
    $(c) \displaystyle\lim_{x \to 0} \frac{\sin x - x}{x^3}$

$(d) \displaystyle\lim_{x \to \infty} (\sqrt{x^2 + x} - x)$
    $(e) \displaystyle\lim_{x \to \infty} \left(\frac{x}{x+1}\right)^x$

(a) 0 (b) 1 (c) $-\frac{1}{6}$ (d) $\frac{1}{2}$ (e) e^{-1}.

(7) If f' is continuous, $f(2) = 0$, and $f'(2) = 6$, evaluate $\displaystyle\lim_{x \to 0} \frac{f(2 + 3x) + f(2 + 5x)}{x}$. (7%) 48.

(8) (a) Let $f'(x) = \sqrt{x}(6 + 5x)$, $f(1) = 10$. Find f. (5%) $f(x) = 4x^{\frac{3}{2}} + 2x^{\frac{5}{2}} + 4$.

   (b) Find the most general antiderivative of $f(x) = 6\sqrt{x} - \sqrt[6]{x}$. (5%) $F(x) = 4x^{\frac{3}{2}} - \frac{6}{7}x^{\frac{7}{6}} + c$.

(9) A box with a square base and open top must have a volume of $32,000$ cm^3. Find the dimensions of the box that minimize the amount of material used. (7%) $40 \times 40 \times 20$.

(10) A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Find the the maximum capacity of such a cup, and when does such maximum capacity occur (i.e. find the corresponding θ)? (7%) Maximum capacity is $\dfrac{2\pi}{9\sqrt{3}}R^3$, which occurs at $\theta = 2\left(1 - \sqrt{\dfrac{2}{3}}\right)\pi$.
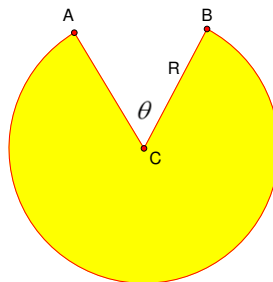
(11) For what value of a is the following equation true? (4%)

$$\lim_{x \to \infty} \left(\frac{x + a}{x - a}\right)^x = e \quad a = \tfrac{1}{2}.$$