Calculus 0314

Quiz 3.

(1) Find the limit. (25%)

(a) $\lim_{x\to 1} \dfrac{\sin(x-1)}{x^2+x-2}$. $\frac{1}{3}$

(b) $\lim_{x\to 0} \dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$. $\frac{1}{4}$

(c) $\lim_{\theta\to \frac{\pi}{3}} \dfrac{\cos\theta - 0.5}{\theta - \frac{\pi}{3}}$. $-\frac{\sqrt{3}}{2}$

(d) $\lim_{t\to 0} \dfrac{t^3}{\tan^3(2t)}$. $\frac{1}{8}$

(e) $\lim_{x\to 0} \dfrac{\sin(3+x)^2 - \sin 9}{x}$. $6\cos 9$

(2) Let f is a one-to-one differentiable function and its inverse function f^{-1} is also differentiable. If $f(2)=3$ and $f'(2)=2$, find $(f^{-1})'(3)$. (6%) $\frac{1}{2}$

(3) A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole? (6%) $\frac{25}{3}$ ft/s

(4) At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 P.M.? (6%) $\frac{720}{13}$ km/h

(5) Find the linearization $L(x)$ of the function $f(x)=\cos x$ at $a=\frac{\pi}{2}$. (6%) $L(x) = -x + \frac{\pi}{2}$

(6) Prove that $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$. (6%)

(7) A window has the shape of a square surmounted by a semicircle. The base of the window is measured as having width 60 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum error possible in computing the area of the window. (5%) $\frac{3\pi}{2} + 12$

(8) Find the derivatives ($\frac{dy}{dx}$). (30%)

(a) $y = xe^{-x^2}$. $e^{-x^2} - 2x^2e^{-x^2}$

(b) $x^2y + xy^2 = 3x$. $\frac{3-y^2-2xy}{x^2+2xy}$

(c) $y = \tan^{-1}\sqrt{x}$. $\frac{x^{-\frac{1}{2}}}{2+2x}$

(d) $y = x\ln x$. $\ln x + 1$

(e) $y = x^x$. $y(\ln x + 1)$

(f) $y = \ln|\sec 5x + \tan 5x|$. $5\sec 5x$

(9) Find $f'(x)$ if it is known that $\frac{d}{dx}[f(2x)] = x^2$. (5%) $f'(x) = \frac{1}{8}x^2$

(10) If f is differentiable at a, where $a>0$, evaluate the following limit in terms of $f'(a)$: $\lim_{x\to a} \dfrac{f(x)-f(a)}{\sqrt{x}-\sqrt{a}}$.

(5%) $\lim_{x\to a} \dfrac{f(x)-f(a)}{\sqrt{x}-\sqrt{a}} = 2\sqrt{a}f'(a)$