Calculus **B** 0314

Quiz 2.

(1) Let

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x \le 0 \\ 2 - x & \text{if } 0 < x \le 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$

(a) Find the numbers at which f is discontinuous. (6%) $\{0\}$.

(b) Find the numbers at which f is not differentiable. (6%) $\{0, 2\}$.

(2) Let

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find the values of m and b that make f differentiable everywhere. (13%) $m = 4$, $b = -4$.

(3) Let $f(x) = \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$. Find the horizontal asymptotes of f. (13%) $y = \pm 3$.

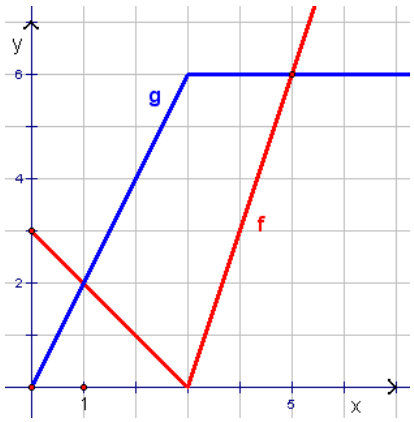
(4) Use the definition to find the derivative of $f(x) = \sqrt{3 - 5x}$. (13%)

$f'(x) = -\dfrac{5}{2}\dfrac{1}{\sqrt{3-5x}} \quad \forall x \in (-\infty, \dfrac{3}{5})$.

(5) Find a function and a number a such that $\lim\limits_{h \to 0} \dfrac{(2+h)^6 - 64}{h} = f'(a)$. (13%) $f(x) = x^6$, $a = 2$.

(6) For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent. (13%) $\left\{\dfrac{-3 \pm \sqrt{6}}{3}\right\}$.

(7) If f and g are the functions whose graphs are shown, let $P(x) = f(x)g(x)$, $Q(x) = \dfrac{f(x)}{g(x)}$, and $C(x) = f(g(x))$. Find (a) $P'(2)$, (b) $Q'(2)$, and (c) $C'(2)$. (13%) (a) -2, (b) $-\frac{3}{8}$, (c) 6.



(8) For $n = 0, 1, 2, \ldots$, that is, n is a nonnegative integer,

$$f_n(x) = \begin{cases} x^n \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) Find n so that $f_n(x)$ is continuous at $x = 0$. (6%) $n \ge 1$.

(b) Find n so that $f_n(x)$ is differentiable at $x = 0$. (6%) $n \ge 2$.