Calculus II 0314

Quiz 6.

(1) (8%) Evaluate the double integral. $\displaystyle\iint_D \frac{2y}{x^2+1}\,dA$, $D=\{(x,y)|0\le x\le 1, 0\le y\le \sqrt{x}\}$. $\frac{1}{2}\ln 2$

(2) (8%) Evaluate $\displaystyle\int_0^1\int_{\sin^{-1}y}^{\frac{\pi}{2}} \cos x\sqrt{1+\cos^2 x}\;dx\;dy$. $\frac{1}{3}(\sqrt{8}-1)$

(3) (8%) Find the volume of the solid described in the following: above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$. $\frac{\pi}{3}(2-\sqrt{2})$

(4) (8%) Evaluate $\displaystyle\int_{\frac{1}{\sqrt{2}}}^1\int_{\sqrt{1-x^2}}^x xy\;dy\;dx+\int_1^{\sqrt{2}}\int_0^x xy\;dy\;dx+\int_{\sqrt{2}}^2\int_0^{\sqrt{4-x^2}} xy\;dy\;dx.$ (Hint: Combine the sum into one double integral). $\frac{15}{16}$

(5) (8%) Find the area of the surface. The part of the sphere $x^2+y^2+z^2=a^2$ that lies within the cylinder $x^2+y^2=ax$ and above the xy-plane. $a^2(\pi-2)$

(6) (8%) Find the area of the part of the surface $z=x^2+y$ that lies above the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$. $\ln(\sqrt{2}+\sqrt{3})+\frac{\sqrt{2}}{3}$

(7) (8%) Evaluate the triple integral. $\displaystyle\iiint_E x\,dV$, where E is bounded by the paraboloid $x=4y^2+4z^2$ and the plane $x=4$. $\frac{16}{3}\pi$

(8) (12%) Rewrite the integral $\displaystyle\int_{-1}^1\int_{x^2}^1\int_0^{1-y} f(x,y,z)\;dz\;dy\;dx$ as an iterated integral in the order $dx\;dy\;dz$ and $dy\;dz\;dx$. $\displaystyle\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz$; $\displaystyle\int_{-1}^1\int_0^{1-x^2}\int_{x^2}^{1-z} f(x,y,z)$ $dy\;dz\;dx$

(9) (8%) Sketch the solid whose volume is given by the integral and evaluate the integral. $\displaystyle\int_0^{\frac{\pi}{6}}\int_0^{\frac{\pi}{2}}\int_0^3 \rho^2\sin\phi\;d\rho\;d\theta\;d\phi$. $\frac{9\pi}{4}(2-\sqrt{3})$

(10) (8%) Use spherical coordinates. Evaluate $\displaystyle\iiint_B (x^2+y^2+z^2)\;dV$, where B is the unit ball $x^2+y^2+z^2\le 1$. $\frac{4}{5}\pi$

(11) (8%) Use spherical coordinates to find the volume of the solid that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$. $\frac{\pi}{8}$

(12) (8%) Evaluate the integral by changing to spherical coordinates. $\displaystyle\int_{-3}^3\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\;dz\;dy\;dx$. $\frac{243}{5}\pi$