Calculus II 0314 Quiz 4.

(1) (9%) Find an equation of the tangent plane to the given surface at the specified point. $z = \sqrt{4 - x^2 - 2y^2}$, $(1, -1, 1)$. $z = -x + 2y + 4$.

(2) (9%) Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$ and use it to approximate the number $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$. $\frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6) + 7$, 6.9914.

(3) (9%) Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick. 16.

(4) (12%) Let $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if}(x, y) \neq (0, 0) \\ 0, & \text{if}(x, y) = (0, 0) \end{cases}$.

(a) Find $f_x(0, 0)$ and $f_y(0, 0)$. 0, 0.

(b) Find the discontinuity of $f_x(x, y)$. $(0, 0)$.

(c) Is f differentiable at $(0, 0)$? No.

(5) (10%)

(a) If $z = f(x, y)$, where f is differentiable, $x = g(t)$, $y = h(t)$, $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, $h'(3) = -4$, $f_x(2, 7) = 6$, and $f_y(2, 7) = -8$, find dz/dt when $t = 3$. 62.

(b) $x - z = \arctan(yz)$. Find $\partial z/\partial x$ and $\partial z/\partial y$. $\frac{\partial z}{\partial x} = \frac{1+y^2z^2}{1+y+y^2z^2}$, $\frac{\partial z}{\partial y} = \frac{-z}{1+y+y^2z^2}$.

(6) Let $z = f(x, y)$, where $x = r\cos\theta$, $y = r\sin\theta$.

(a) (4%) Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$.

(b) (9%) Find $\frac{\partial^2 z}{\partial r^2}$, $\frac{\partial^2 z}{\partial \theta^2}$ and $\frac{\partial^2 z}{\partial r \partial \theta}$.

(7) (10%)

(a) Find the directional derivative of the function $g(x, y, z) = (x + 2y + 3z)^{3/2}$ at the point $(1, 1, 2)$ in the direction of the vector $\mathbf{v} = 2\mathbf{j} - \mathbf{k}$. $\frac{9}{2\sqrt{5}}$.

(b) Find the maximum rate of change of $f(x, y) = \sin(xy)$ at the point $(1, 0)$ and the direction in which it occurs. 1, $(0, 1)$.

(8) (10%) Let f be a function of two variables that has continuous partial derivatives and consider the points $A(1, 1)$, $B(3, 3)$, $C(1, 7)$, and $D(6, 15)$. The directional derivative of f at A in the direction of the vector $\overrightarrow{AB}$ is 3 and the directional derivative at A in the direction of $\overrightarrow{AC}$ is 26. Find the directional derivative of f at A in the direction of the vector $\overrightarrow{AD}$. $\frac{15\sqrt{2}+234}{\sqrt{221}}$.

(9) (9%) Find the points on the hyperboloid $x^2 - y^2 + 2z^2 = 1$ where the normal line is parallel to the line that joins the points $(3, -1, 0)$ and $(5, 3, 6)$. $\pm(\frac{\sqrt{6}}{3}, -\frac{2\sqrt{6}}{3}, \frac{\sqrt{6}}{2})$.

(10) (9%) Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $(-1, 1, 2)$. $\frac{x+1}{5} = \frac{y-1}{8} = \frac{z-2}{6}$.