Calculus 0314

Quiz 2.

(1) Determine whether the series is absolutely convergent, conditionally convergent or divergent.

(a) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{5+n}$ (b) $\sum_{n=1}^{\infty}\frac{\cos(\frac{n\pi}{3})}{n!}$ (c) $\sum_{n=1}^{\infty}\frac{2\cdot 4\cdot 6\cdot\cdots\cdot(2n)}{n!}$ (20%)

(d) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$.

(a) divergent, (b) absolutely convergent, (c) divergent, (d) conditionally convergent.

(2) Find the radius of convergence and interval of convergence. (15%)

(a) $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$ (b) $\sum_{n=1}^{\infty}(-1)^n\frac{(x+2)^n}{n2^n}$ (c) $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^n$.

(a) radius= 1, interval= $[-1,1)$; (b) radius= 2, interval= $(-4,0]$; (c) radius= 4, interval= $(-4,4)$.

(3) Suppose $\sum_{n=0}^{\infty}c_n(x-2)^n$ converges when $x=6$ and diverges when $x=-4$. What can be said about the convergence or divergence of the following series? (8%)

(a) $\sum_{n=1}^{\infty}c_n$ C (b) $\sum_{n=1}^{\infty}(-1)^n c_n$ C (c) $\sum_{n=0}^{\infty}c_n 7^n$ D (d) $\sum_{n=0}^{\infty}c_n 8^n$. D

(4) Suppose the series $\sum c_n x^n$ and $\sum d_n x^n$ have, respectively, radius of convergence 2 and 3.

(a) What is the radius of $\sum(c_n+d_n)x^n$? (5%) 2.

(b) What is the radius of $\sum c_n x^{2n}$? (5%) $\sqrt{2}$.

(5) Find a power series representation for the function and determine the radius of convergence.

(a) $f(x)=\ln(5-x)$ (b) $f(x)=\frac{x^2}{(1+x)^2}$. (12%)

(a) $f(x)=\ln 5-\sum_{n=1}^{\infty}\frac{x^5}{n\cdot 5^n}$, and radius $=5$.

(b) $f(x)=\sum_{n=0}^{\infty}(-1)^n(n+1)x^{n+2}$, and radius $=1$.

(6) (a) Find $\sum_{n=1}^{\infty}nx^{n-1}=?$ as $|x|<1$. (Hint: $(x^n)'=nx^{n-1}$.) (5%) $\frac{1}{(1-x)^2}$.

(b) $\sum_{n=1}^{\infty}nx^n=?$ as $|x|<1$. (5%) $\frac{x}{(1-x)^2}$.

(c) $\sum_{n=1}^{\infty}\frac{n}{2^n}=?$ (5%) 2.

(7) Find the Maclaurin series of $f(x)=\cos x$. (Assume that f has a power series expansion). (6%)

$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$.

(8) Use the binomial series to expand the function $f(x)=\frac{x}{\sqrt{4+x^2}}$ and state its radius of convergence. (6%) $f(x)=\frac{x}{2}+\sum_{n=1}^{\infty}(-1)^n\frac{1\cdot 3\cdot\cdots\cdot(2n-1)}{2^{3n+1}n!}x^{2n+1}$, and radius=2.

(9) (a) Find the Maclaurin series of $f(x)=\sqrt{1+x^2}$. (5%)

(b) Evaluate $f'(10)$. (3%) Should be corrected by $f^{(10)}(0)$.

(a) $1+\frac{x^2}{2}+\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot\cdots\cdot(2n-3)}{2^n n!}x^{2n}$, (b) $f'(10)=\frac{10}{\sqrt{101}}$, $f^{(10)}(0)=99,225$.