Calculus 0314

Quiz 1.

(1) True or False(T or F). No explanations are needed. (30%)

F 1. If $\lim\limits_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

F 2. The series $\sum_{n=1}^{\infty} n^{-\cos 1}$ is convergent.

T 3. If $\lim\limits_{n\to\infty} a_n = L$, then $\lim\limits_{n\to\infty} a_{2n+1} = L$.

F 4. If $\lim\limits_{n\to\infty} a_n = L$, $n \in \mathbb{N}$, then $\lim\limits_{x\to\infty} a_x = L$, $x \in \mathbb{R}$.

F 5. If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

T 6. If $-1 < \alpha < 1$, then $\lim\limits_{n\to\infty} \alpha^n = 0$.

F 7. If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} |a_n|$ is convergent.

F 8. If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent.

T 9. If $\{a_n\}$ is decreasing and $a_n > 0$ for all n, then $\{a_n\}$ is convergent.

F 10. If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms and $\sum_{n=1}^{\infty} b_n$ is convergent. If $\lim\limits_{n\to\infty} \dfrac{b_n}{a_n} = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

T 11. If $a_n > 0$ and $\lim\limits_{n\to\infty} na_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

T 12. If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} \sin(a_n)$ is convergent.

T 13. If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both convergent with positive terms, then $\sum_{n=1}^{\infty} a_n b_n$ is convergent.

F 14. The series $1 - \dfrac{1}{2^2} + \dfrac{1}{3} - \dfrac{1}{4^2} + \cdots + \dfrac{1}{2n-1} - \dfrac{1}{(2n)^2} + \cdots$ is convergent.

T 15. Let $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n$. Then $a_n > 0$ for all n.

(2) (i) Find the values of x for which the series $\displaystyle\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$ converges. (ii) Find the sum of the series for these values of x. (10%) (i) $x \in \mathbb{R}$. (ii) $\frac{2}{2-\cos x}$.

(3) If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n-1}{n+1}$. Find a_n and $\sum_{n=1}^{\infty} a_n$. (10%) $\frac{2}{n(n+1)}$, and 1.

(4) Find the values of p for which the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ is convergent. (10%) $p > 1$.

(5) Find all positive values of b for which the series $\displaystyle\sum_{n=1}^{\infty} b^{\ln n}$ converges. (10%) $b < e^{-1}$.

(6) Determine whether the series converges or diverges(Explain briefly why.) (30%)

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{2 + (-1)^n}{n\sqrt{n}}$ C (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1}$ C (iii) $\displaystyle\sum_{n=1}^{\infty} \sin(\frac{1}{n})$ D

(iv) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$ D (v) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$ D (vi) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{\ln n}$ D.