A function $f: X \subseteq \mathbb{R} \to \mathbb{R}$ takes a real number and returns another real number. Single Variable Scalar Function The graph of f is something that "lives" in \mathbb{R}^2 . It consists of points (x,y) such that y=f(x). That is, Graph $f=\{(x,f(x))\mid x\in X\}=\{(x,y)\mid x\in X,y=f(x)\}.$ The level curve at height c of f is the curve in \mathbb{R}^2 defined by the equation f(x,y)=c, where c is a constant. Symbolized, level curve at height $c=\{(x,y)\in\mathbb{R}^2\mid f(x,y)=c\}$. A function $f: X \subseteq \mathbb{R}^2 \to \mathbb{R}$ takes an ordered pair of real number and returns another real number. Contour Curve at height c of f is the curve in \mathbb{R}^3 defined by the two equations z=f(x,y) and z=c. Symbolized, contour curve at height $c=\{(x,y,z)\in\mathbb{R}^3\mid z=f(x,y)=c\}$. Two Variable Scalar Function The graph of f is something that "lives" in \mathbb{R}^3 . It consists of points (x,y,z) such that z=f(x,y). -Graph That is, Graph $f = \{(x, y, f(x, y)) \mid x, y \in X\} = \{(x, y, z) \mid x, y \in X, z = f(x, y)\}.$ Section The section of the graph of f by the plane $\begin{cases} x = c \text{ (where } c \text{ is a constant) is the set of points } (x, y, z), \text{ where } z = f(x, y) \text{ and } x = c. \\ y = c \text{ (where } c \text{ is a constant) is the set of points } (x, y, z), \text{ where } z = f(x, y) \text{ and } y = c. \end{cases}$ Visualizing Scalar Functions (Using Graph) The level set at height c of such a function is defined by Level set at height $c = \{\mathbf{x} \in \mathbb{R}^n \mid f(\mathbf{x}) = c\}$. Functions of More Variables $egin{aligned} ext{If } f: X \subseteq \mathbb{R}^n &
ightarrow \mathbb{R} ext{ is a scalar function of } n ext{ variable, then the } \mathbf{graph} ext{ of } f ext{ is the subset of } \mathbb{R}^{n+1} ext{ given by } \ ext{Graph } f = \{(\mathbf{x}, f(\mathbf{x}) \mid \mathbf{x} \in X\} = \{(x_1, \dots, x_n, x_{n+1}) \mid (x_1, \dots, x_n) \in X, x_{n+1} = f(x_1, \dots, x_n)\}. \end{aligned}$ While the graph of f is a subset of \mathbb{R}^{n+1} , a level set of f is a subset of \mathbb{R}^n . This makes it possible to get some geometric insight into graphs of functions of three variables, even though we can't actually visualize them. The graph of a function of two variables does describe a surface in the "level set" sense. If a surface happens to be given by an equation of the form z=f(x,y) for some appropriate function $f\colon X\subseteq\mathbb{R}^2\to\mathbb{R},$ then we can move z to the opposite side, obtaining f(x,y)-z=0. If we define a new function F of 神充 three variables by F(x,y,z)=f(x,y)-z, then the graph of f is precisely the level set at height 0 of F. Conclusion: The graph of a function of two variables is a surface in \mathbb{R}^3 and is a "level set" of a function of three variables. However, not all level sets of functions of three variables are graphs of functions of two variables. Let $f:X\subseteq \mathbb{R}^n o \mathbb{R}$ be a function. Then to say $\lim_{\mathbf{x} o \mathbf{a}} f(\mathbf{x}) = L$ means that given any arepsilon > 0, 岩極限值存在則唯一 you can find a $\delta > 0$ such that if $\mathbf{x} \in X$ and $0 < \|\mathbf{x} - \mathbf{a}\| < \delta$, then $\|f(\mathbf{x}) - L\| < \varepsilon$. 為了幫助你理解,以下是其他微積分用書常見的雙變數函數極限定義。 Let f be a function defined on an open disk (圓盤) centered at (a,b) [except possibly at (a,b)]. Definition We say that the limit of f(x,y) as (x,y) approaches (a,b) is L and we write $\lim_{(x,y) o (a,b)} f(x,y) = L$ $\text{if for all $\epsilon > 0$, there is a $\delta > 0$ such that } |f(x,y)-L| < \epsilon \text{ whenever } 0 < \sqrt{(x-a)^2+(y-b)^2} < \delta \,.$ Limit of Scalar Function of Two Variable 因為其他教科書常用雙變數純量函數舉例,故以此為例。 $\lim_{(x,y) o(a,b)} \lim_{ ext{along path } C_1} f(x,y)
eq \lim_{(x,y) o(a,b) ext{along path } C_2} f(x,y) \ ,$ $ext{Does } \lim_{(x,y) o(0,0)} rac{xy^2}{x^2+y^4} ext{ exists?}$ 證明不存在 $\lim_{(x,y) o (a,b)} f(x,y)$ does not exits. For the path x=0: $\lim_{(x,y) o(0,0)} \dfrac{xy^2}{x^2+y^4}=0$ $\lim_{\mathbf{x} o \mathbf{a}} (f \pm g)(\mathbf{x}) = L \pm M$ Though f has the same limiting value along every line through the origin. $\lim_{\mathbf{x} o \mathbf{a}} k f(\mathbf{x}) = k L$ $\text{Let } f,g\text{:}\ X\subseteq\mathbb{R}^n\to\mathbb{R} \text{ be scalar-valued functions, } \lim_{\mathbf{x}\to\mathbf{a}}(fg)(\mathbf{x})=LM$ and let $k \in \mathbb{R}$ be a scalar. $\lim_{\mathbf{x} o \mathbf{a}} ig[f(\mathbf{x})ig]^n = L^n$... The limit doesn't exist. Limit Laws $\lim_{\mathbf{x} o \mathbf{a}} f(\mathbf{x}) = L \ \lim_{\mathbf{x} o \mathbf{a}} g(\mathbf{x}) = M$ $\lim_{\mathbf{x} o \mathbf{a}} (rac{f}{g})(\mathbf{x}) = rac{L}{M} \quad (M
eq 0)$ $\lim_{\mathbf{x} o\mathbf{a}}\sqrt[n]{f(\mathbf{x})}=\sqrt[n]{L} ext{ where }n\in\mathbb{R}$ $\text{If } n \text{ is even, we assume that } \lim_{\mathbf{x} \to \mathbf{a}} f(\mathbf{x}) \geq 0.$ Let $A\subseteq \mathbb{R}^n$ be a set, and $f,g:A\to \mathbb{R}^m$ be scalar functions. $\text{Suppose that } x_0 \in A, f(\mathbf{x}) = g(\mathbf{x}) \text{ for all } \mathbf{x} \in A - \{\mathbf{x}_0\}, \text{ and } \lim_{\mathbf{x} \to \mathbf{x}_0} f(\mathbf{x}) \text{ exists.}$ $ext{Then } \lim_{\mathbf{x} o \mathbf{x}_0} g(\mathbf{x}) ext{ exists and } \lim_{\mathbf{x} o \mathbf{x}_0} g(\mathbf{x}) = \lim_{\mathbf{x} o \mathbf{x}_0} f(\mathbf{x}).$ 求 $\lim_{(x,y) o(0,0)}x\sin(x+y)$ Theorem $oldsymbol{-}$ Squeeze Theorem $oldsymbol{\mathrm{and}}$ $oldsymbol{\lim}_{(x,y) o(a,b)} f(x,y) = \lim_{(x,y) o(a,b)} h(x) = L \Rightarrow \lim_{(x,y) o(a,b)} g(x) = L$ $\lim_{(x,y) o(0,0)}|x\sin(x+y)|=0\Rightarrow \lim_{(x,y) o(0,0)}x\sin(x+y)=0$ Let $f: X \subseteq \mathbb{R}^n \to \mathbb{R}$ and let $\mathbf{a} \in X$. Then f is said to be continuous at \mathbf{a} if $\lim_{\mathbf{x} \to \mathbf{a}} f(\mathbf{x}) = f(\mathbf{a})$. If f is continuous at all points of its domain X, then we simply say that f is continuous. If $f,g:X\subseteq\mathbb{R}^n\to\mathbb{R}$ that are continuous at $\mathbf{a}\in X$ and c is a constant, then the following functions are also continuous at \mathbf{a} : $\boxed{1.\ f\pm g} \ \ \ 2.\ cf \ \ \ 3.\ fg \ \ \ 4.\ rac{J}{a} \ \ ext{if} \ \ g(a,b)
eq 0$

多變數純量函數

If g is continuous at the point $\mathbf{x_0}$ and f is continuous at the number $g(\mathbf{x_0})$,

then the composition $f \circ g$ is continuous at the point \mathbf{x}_0 .

極限與連續

在畫等高線圖時, k之間的差距通常取相同

左邊是 Contour Curves;右邊是 Level Curves