§6-5 Average Value of a Function

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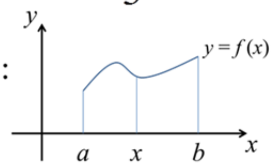
平均：
- 離散：$\dfrac{x_1 + x_2 + x_3}{3}$
- 連續：



*公式：

The average f_{ave} of a function $y = f(x)$ on $[\, a , b \,]$ is

$$f_{ave} = \frac{\int_a^b f(x)dx}{b-a}.$$

> *註記：
>
> $(b-a) f_{ave}$ = 長方形面積 = 曲線底下由b到a的(有向)面積 = $\int_a^b f(x)dx$

*The Mean Value Theorem for Integrals：

If f is continuous on $[\, a , b \,]$, then ∃ a c in $[\, a , b \,]$ s.t.

$$f(c) = f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx.$$

*Show that the average velocity of a car over a time interval $[\, t_1, t_2 \,]$ is the same as the average of its velocities during the trip.(平均速度 ＝ 瞬間速度的平均值)

Proof：

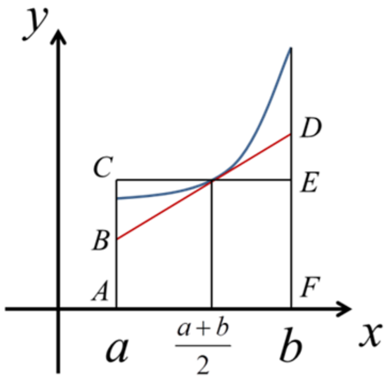
$$平均速度 = \frac{s(t_2)-s(t_1)}{t_2-t_1}, \text{ where } s(t) = \text{position function.}$$

$$瞬間速度的平均值 = \frac{\int_{t_1}^{t_2} s'(t)dt}{t_2-t_1} = \frac{s(t_2)-s(t_1)}{t_2-t_1}.$$

Example 1：

Use the diagram to show that if f is concave upward on [a , b], then

$$f_{ave} > f\left(\frac{a+b}{2}\right).$$



Solution：

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$

$$> \frac{1}{b-a} \times (\text{area of trapezoid } ABDF)$$

$$= \frac{1}{b-a} \times (\text{area of rectangle } ACEF)$$

$$= \frac{1}{b-a} \times \left[f\left(\frac{a+b}{2}\right) \times (b-a) \right]$$

$$= f\left(\frac{a+b}{2}\right).$$