# §3-2 The Product and Quotient Rules

＊一些微分法則：

**公式：**

i. Product Rule

$(fg)' = f'g + fg'$

ii. Quotient Rule

$(\frac{f}{g})' = \frac{f'g - fg'}{g^2}$

Proof of (i)：

已知：$f$ and $g$ are differentiable.

求證：$(fg)' = f'g + fg'$.

$$(fg)'(x) = \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\frac{g(x+h)[f(x+h) - f(x)] + f(x)[g(x+h) - g(x)]}{h}$$

(分子同時“+”同時“-” $f(x)g(x+h)$)

Since $f$ and $g$ are differentiable, and, hence continuous, the limit above exists and equals to $f'(x)g(x) + f(x)g'(x)$.

**Remark**：

iii. $(fgh)' = f'gh + fg'h + fg'h$

**Question**：Why formulas such as (i) & (iii) are “reasonable” ?
(考慮當 $f, g, h$ 是多項式時的特例，為何(i)和(iii)是合理的)



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**Example 1**：

Let $f(x)=\dfrac{x-3x\sqrt{x}}{\sqrt{x}}$ , find $f'(x)$.

**Solution**：

$$f(x)=x^{\frac{1}{2}}-3x \Rightarrow f'(x)=\frac{1}{2}x^{-\frac{1}{2}}-3$$

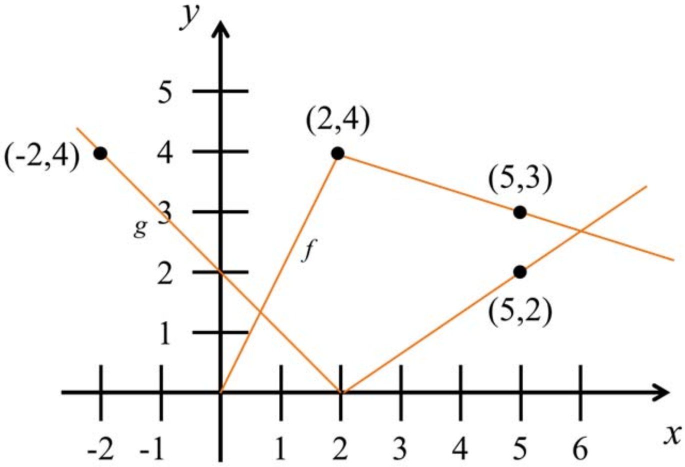
**Example 2**：

Let $h(2)=4, h'(2)=-3$, find $\left.\dfrac{d}{dx}\left(\dfrac{h(x)}{x}\right)\right|_{x=2}$

**Solution**：

$$\left.\frac{d}{dx}\left(\frac{h(x)}{x}\right)\right|_{x=2}=\left.\frac{h'(x)x-h(x)}{x^2}\right|_{x=2}=\frac{-3\times 2-4}{4}=-\frac{5}{2}$$

**Example 3**：




Let $u(x)=f(x)g(x)$, $v(x)=\dfrac{f(x)}{g(x)}$.

$(a)$ Find $u'(1)$.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(b) Find $v'(5)$.

**Solution**：

$$(a)\quad u'(1) = [f(1)g(1)]' = f'(1)g(1) + f(1)g'(1) = 2\times 1 + 2\times(-1) = 0.$$

$$(b)\quad v'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{[g(5)]^2} = \frac{\left(-\frac{1}{3}\right)\times 2 - 3\times\frac{2}{3}}{2^2} = -\frac{2}{3}.$$