§2-9 The Derivative As a Function

Notations： $f'(x) = y' = \dfrac{dy}{dx} = \dfrac{df}{dx} = Df(x) = D_x f(x)$.

Example 1： $f(x) = x^3 - x$. Compute $f'(x)$ by definition.

Solution：

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0}\frac{[(x+h)^3 - (x+h)] - (x^3 - x)}{h}$$
$$= 3x - 1$$

Example 2： $f(x) = \sqrt{x-1}$. Compute $f'(x)$ by definition.

Solution：

$$f'(x) = \lim_{h\to 0}\frac{\sqrt{(x+h)-1} - \sqrt{x-1}}{h}$$
$$= \lim_{h\to 0}\frac{[(x+h)-1] - (x-1)}{h(\sqrt{(x+h)-1} + \sqrt{x-1})}$$
$$= \lim_{h\to 0}\frac{1}{\sqrt{(x+h)-1} + \sqrt{x-1}}$$
$$= \frac{1}{2\sqrt{x-1}}$$

Example 3：$f(x) = x\,|\,x\,|$. Does f have a derivative at $x = 0$?

Solution：

$$f'(0) = \lim_{x\to 0}\frac{x\,|\,x\,|}{x} = \lim_{x\to 0}|\,x\,| = 0 .$$