Basic Order Statistics

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The subject of order statistics

Notations

• If the random variables X_1, X_2, \ldots, X_n are arranged in ascending order of magnitude and then written as:

$$X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)},$$

we call $X_{(i)}$ the *i*th order statistic.

- The **subject** of order statistics deals with the properties and applications of these ordered random variables, as well as functions involving them.
 - Examples of functions of ordered random variables are

the range
$$W_n = X_{(n)} - X_{(1)}$$

and

the extreme deviate
$$D_n = X_{(n)} - \bar{X}$$
,

where \overline{X} is the average of X_1, \ldots, X_n .

The subject of order statistics

- An example of applications is the situation that if an experiment fails when k-out-of-n outcomes are beyond a threshold, then (n k + 1)th order statistic can give the probability of experimental failure.
- One may also give a better estimate with small number of samples, if outliers (extremes) are removed.