

§14.1 Functions of Several Variables

* 函數型： $f: R^2 \rightarrow R$ or $f: R^3 \rightarrow R$

(1) Domain (2) Range (3) Graph

(4) Level curves(等高線) or Surfaces.

Example 1 :

$$f(x, y) = \sqrt{9 - x^2 - y^2} = z$$

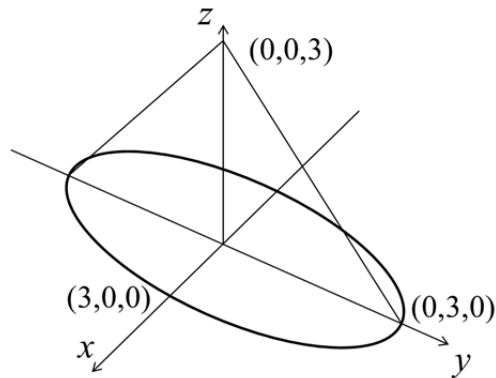
(1) Domain (2) Range (3) Graph (4) Level curves

Solution :

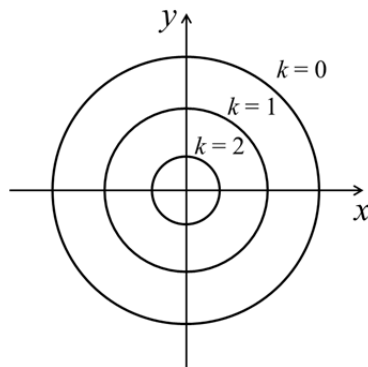
(1) Domain : $9 - x^2 - y^2 \geq 0; \{(x, y) : x^2 + y^2 \leq 9\}$

(2) Range : $\{z : 0 \leq z \leq 3\}$

(3) Graph :



(4) Level curve : $\sqrt{9 - x^2 - y^2} = k$



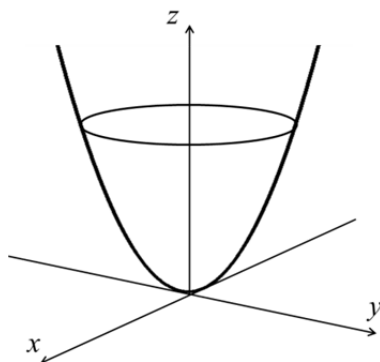
Example 2 :

$$f(x, y) = 4x^2 + y^2 = z$$

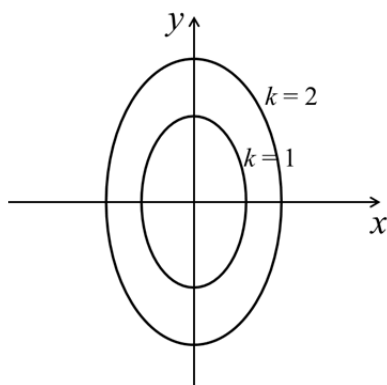
- (1) Domain (2) Range (3) Graph (4) Level curves

Solution :

- (1) Domain : R^2
(2) Range : $\{z : z \geq 0\}$
(3) Graph :



(4) Level curve : $4x^2 + y^2 = k \Rightarrow \frac{x^2}{\left(\frac{\sqrt{k}}{2}\right)^2} + \frac{y^2}{(k)^2} = 1$



Example 3 :

$$f(x, y, z) = x^2 + y^2 + z^2 = u$$

- (1) Domain (2) Range (3) Graph (4) Level surfaces

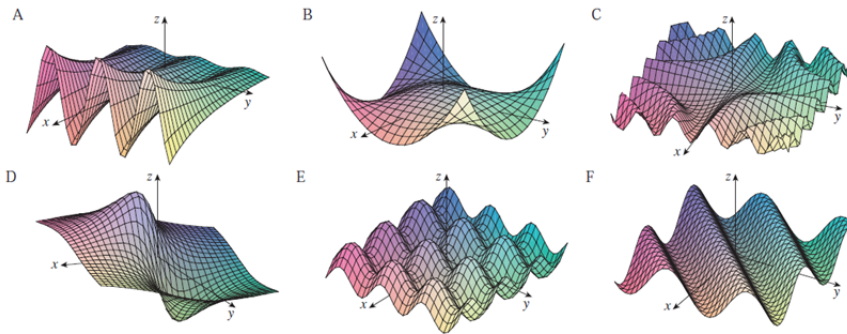
Solution :

- (1) Domain : R^3
(2) Range: $\{u : u \geq 0\}$
(3) 四度空間
(4) Level surfaces : $x^2 + y^2 + z^2 = k$

Example 4 :

Match the function with its graph (labeled A-F).

- i. $z = \sin(xy)$ ii. $z = e^x \cos y$
iii. $z = \sin(x - y)$ iv. $z = \sin x - \sin y$
v. $z = (1 - x^2)(1 - y^2)$ vi. $z = \frac{x - y}{1 + x^2 + y^2}$



Solution :

- i. C
ii. A
iii. F
iv. E
v. B
vi. D