§13.2 Derivatives and Integrals of Vector Functions

Definition：

Let $r(t)=\langle f(t),g(t),h(t)\rangle$.

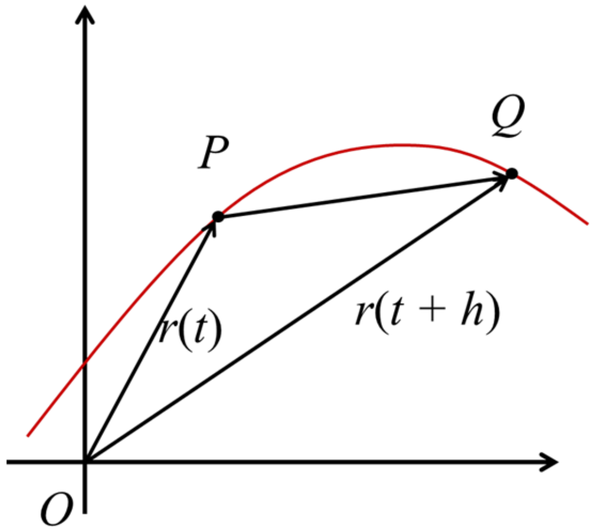
Then $\Delta r(t)=\langle \Delta f(t),\Delta g(t),\Delta h(t)\rangle$

Here $\Delta = \lim_{t\to a}$, $\frac{d}{dt}$, $\int$

(極限) (微分) (積分)

∗ $r'(t)$ = the tangent vector to the curve defined by r at the point P.

(過 P 點的 r 曲線的切向量) = 在 t 時間的(瞬間)速度向量



$$\overline{PQ}=\overline{OQ}-\overline{OP}=r(t+h)-r(t)=\text{割向量}$$

$$\Rightarrow \lim_{h\to 0}\frac{r(t)-r(t+h)}{h}=r'(t)=\text{過}P\text{點的切向量}.$$

Example 1：

Let $x=t^5, y=t^4, z=t^3, t\in R$. Find the equation of tangent to the curve at (1,1,1).

Solution：

$$r'(t)=\langle 5t^4,4t^3,3t^2\rangle_{t=1}=\langle 5,4,3\rangle.$$

Example 2：

Let $r(t)$ = < cos t, sin t, t >. Find

(i) Velocity when $t = 2\pi$ (ii) Distance traveled over [0, 2π].

Solution：

(i). $v(t) = r'(t) = \langle -\sin t, \cos t, 1 \rangle_{t=2\pi} = \langle 0,1,1 \rangle$

$|v(t)| =$ 速度(speed) $= \sqrt{2}$.

(ii). $d = \int_0^{2\pi} |v(t)| dt = 2\pi\sqrt{2} = 2\sqrt{2}\pi$.

＊Differentiation Rules

(i). $\frac{d}{dt}\left(u(t)\cdot v(t)\right) = u'(t)\cdot v(t) + u(t)\cdot v'(t)$

(ii). $\frac{d}{dt}\left(u(t)\times v(t)\right) = u'(t)\times v(t) + u(t)\times v'(t)$

＊ $\frac{d}{dt}|r(t)| = \frac{r'(t)\cdot r(t)}{|r(t)|}$

Proof：

$|r(t)|^2 = r(t)\cdot r(t)$

$\Rightarrow 2|r(t)|\frac{d}{dt}|r(t)| = 2r'(t)\cdot r(t)$

$\Rightarrow \frac{d}{dt}|r(t)| = \frac{r'(t)\cdot r(t)\cdot}{|r(t)|}$

Example 3：

Let $f(x, y) = x - y^2$. Find the tangent line to the level curve $f(x, y) = 2$ at the point (3, -1).

Solution：

Let $y = t \Rightarrow 2 + t^2$

$r(t) = (2+t^2, t, 0) \Rightarrow r'(t) = (2t, 1, 0)$

由(3,-1)可知$t = -1$代入

$r'(-1) = (-2,1,0) \Rightarrow x = -2t+3, y = t-1$

$\Rightarrow x + 2y = -2t + 3 + 2(t-1) = 1$