

§13.1 Vector Functions

Functions :

- (i). $f: R \rightarrow R$
- (ii). $f: R \rightarrow R^m$
- (iii). $f: R^n \rightarrow R$
- (iv). $f: R^n \rightarrow R^m$

- (i) 型的函數之微分和積分是我們之前微積分課程學的東西。
- (ii) 和 (iii) 型函數的微分和積分是這學期剩下的時間要學習的。
- (ii) 型函數的微分和積分是 (i) 型函數的直接推廣；
- (iii) 型較為複雜(why?)
- (iv) 型函數的微分在高微的課程會學到。

1. Vector Functions or Vector – valued Functions

(ii) 型函數即積 Vector Functions.

$$r(t) = \langle f(t), g(t), h(t) \rangle = f(t)i + g(t)j + h(t)k, \quad f, g, h \text{ 是實函數.}$$

2. Space curve : If f, g, h are continuous real-valued functions, then the orbit of

$$r(t) = \langle f(t), g(t), h(t) \rangle \text{ is called a space curve.}$$

Example 1 :

$$\text{Sketch the curve } r(t) = \langle 1+t, 2+5t, -1+6t \rangle$$

Solution :

It is a straight line.

Example 2 :

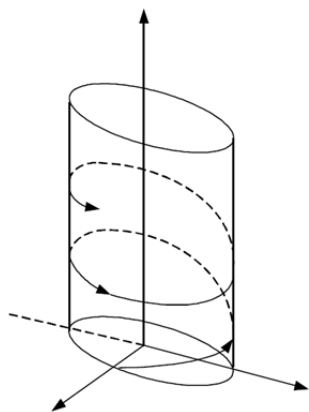
$$\text{Sketch the curve } r(t) = \langle \cos t, \sin t, t \rangle$$

Solution :

$$x^2 + y^2 = 1$$

$$z = t$$

This graph is called “helix”(螺旋).



Example 3 :

Find a vector function that represents the curve of intersection of the two surface : the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

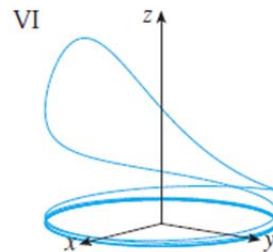
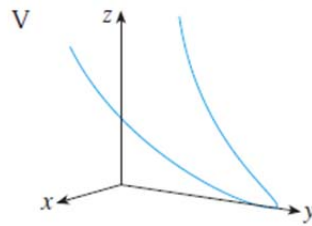
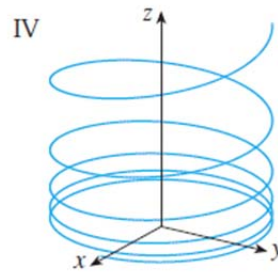
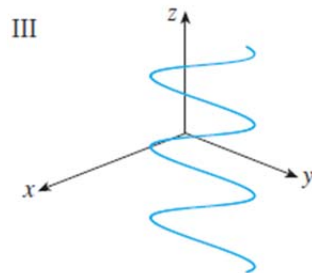
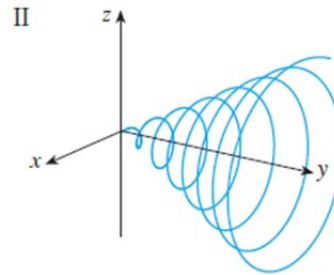
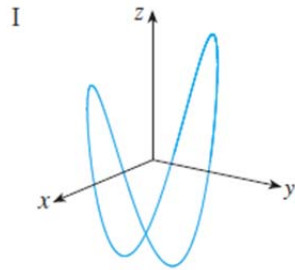
Solution :

$$(1 + y)^2 = \sqrt{x^2 + y^2}$$
$$\Rightarrow x^2 = 1 + 2y$$

$$r(t) = \left(t, \frac{t^2 - 1}{2}, \frac{t^2 + 1}{2} \right)$$

Example 4 :

Match the parametric equation with the graphs (labeled I-VI). Given reasons for your choices.



1. $x = t \cos t, y = t, z = t \sin t, t \geq 0$
2. $x = \cos t, y = \sin t, z = \frac{1}{(1+t^2)}$
3. $x = t, y = \frac{1}{(1+t^2)}, z = t^2$
4. $x = \cos t, y = \sin t, z = \cos 2t$
5. $x = \cos 8t, y = \sin 8t, z = e^{0.8t}, t \geq 0$
6. $x = \cos^2 t, y = \sin^2 t, z = t$

Solution :

1. II
2. VI
3. V
4. I
5. IV
6. III