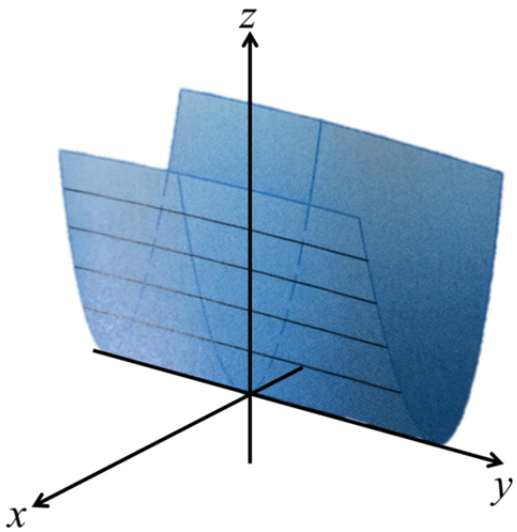
§12.6 Cylinders and Quadrate Surface

Definition：

1. Cylinders：A surface consisting of all lines (called rulings) that parallel to a given line and pass through a given plane curve.

   Example：

   $z = x^2$ (Parabolic cylinder)



   i. The rulings of the cylinder are parallel to the y-axis.

   ii. The rulings of the cylinder pass through a given plane curve $z = x^2$ and $y = 0$.

   iii. If one of the variables x, y or z is missing from the equation of a surface, then the surface is a cylinder.

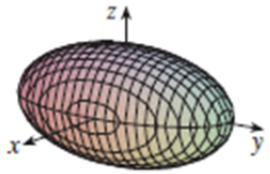
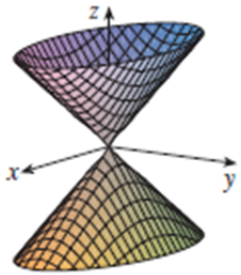
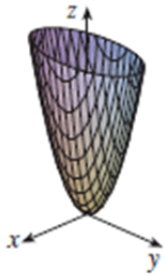
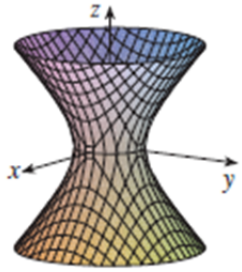
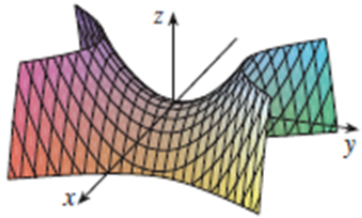
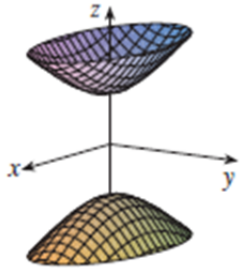
2. Quadratic Surface vs. Quadratic Curves

   2^{nd} – degree equation in 3 (resp. z) variable x, y and z (resp. x and y)

   高中：Quadratic Curves $\xrightarrow[\text{(矩陣)}]{\text{平移和轉軸}}$ $\begin{cases} Ax^2 + By^2 + J = 0 \\ Ax^2 + Hy = 0 \end{cases}$

   大學：Quadratic Surfaces $\xrightarrow[\text{(矩陣)}]{\text{平移和轉軸}}$ $\begin{cases} Ax^2 + By^2 + Cz^2 + J = 0 \\ Ax^2 + By^2 + Iz = 0 \end{cases}$

3. Standard Quadratic Surfaces

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ All traces are ellipses. If $a=b=c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x=k$ and $y=k$ are hyperbolas if $k\neq 0$ but are pairs of lines if $k=0$.
Elliptic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c<0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Horizontal traces in $z=k$ are ellipses if $k>c$ or $k<-c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

Example 1：

Match the equation with its graph (labeled I - VIII). Give reason for your choice.

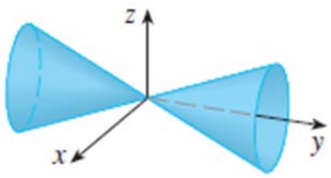
1. $x^2+4y^2+9z^2=1$　2. $9x^2+4y^2+z^2=1$

3. $x^2-y^2+z^2=1$　4. $-x^2+y^2-z^2=1$

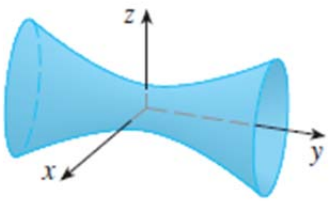
5. $y=2x^2+z^2$　6. $y^2=x^2+2z^2$

7. $x^2+2z^2=1$　8. $y=x^2-z^2$

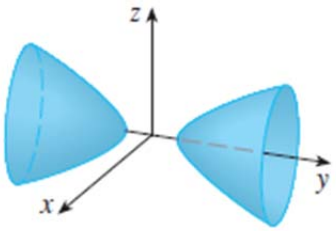
I



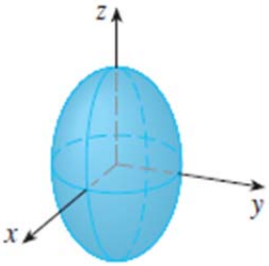
II



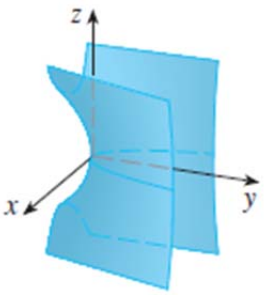
III



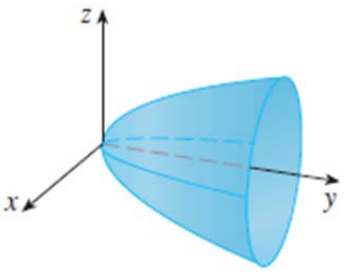
IV



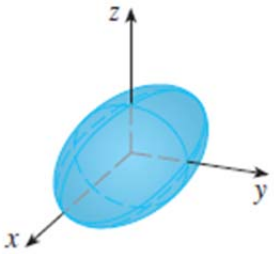
V



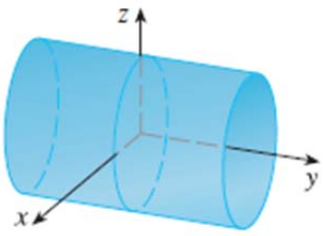
VI



VII



VIII



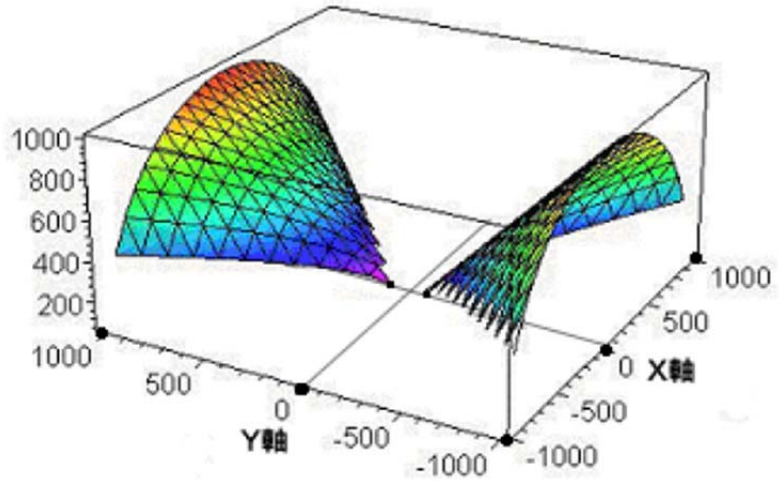
Example 2：

Identify the trace cross section of the surface $x = 2y^2 + 3z^2$ in the plane $x = 1$.

(1) Ellipse but not circle
(2) Parabola
(3) Hyperbola
(4) Circle
(5) Two Parallel straight lines
(6) Two intersection straight lines
(7) Point
(8) Straight Line

Example 3：

Given the following graph：



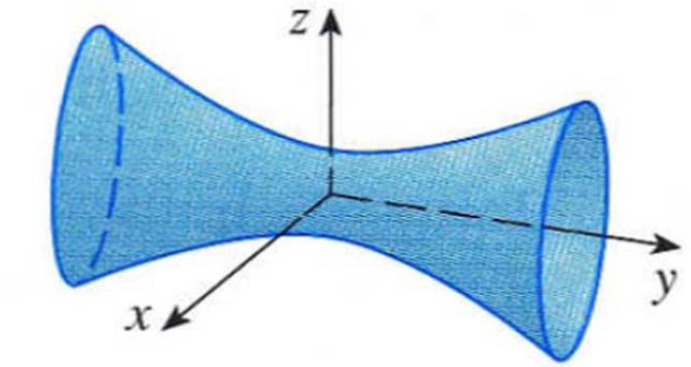
Which equation in the following matches its graph as above?

A) $9x^2 + 4y^2 + z^2 = 1$ B) $y = x^2 - z^2$

C) $x^2 - y^2 + z^2 = 1$ D) $-x^2 + y^2 - z^2 = 1$

Example 4：

The graph



is the equation of

A) $x^2 + 4y^2 + 9z^2 = 1$ B) $x^2 - y^2 + z^2 = 1$

C) $-x^2 + y^2 - z^2 = 1$ D) $y^2 = x^2 + 2z^2$