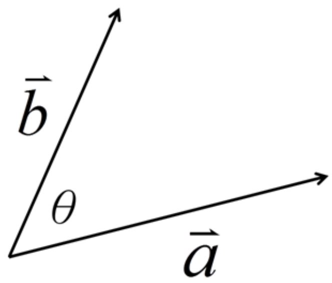
§12.3 The Dot Product

∗Dot product(a scalar) $\vec{a}\cdot\vec{b}$

Let $\vec{a}=<a_1,a_2,a_3>$, $\vec{b}=<b_1,b_2,b_3>$.

1. 代數意義：$\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$

   幾何意義：$\vec{a}\cdot\vec{b}=|\vec{a}|\times|\vec{b}|\times\cos\theta$

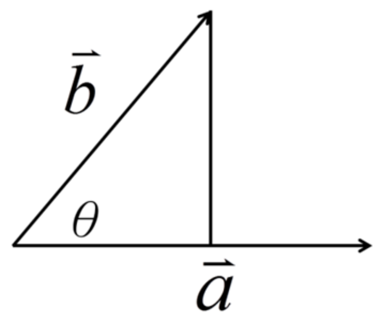


2. $\vec{a}\perp\vec{b}\Leftrightarrow\vec{a}\cdot\vec{b}=0.$

3. $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}.$

4. Scalar projection of $\vec{b}$ onto $\vec{a}$:

   $=|\vec{b}|\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}.$



5. Vector projection of $\vec{b}$ onto $\vec{a}$:

   $=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\dfrac{\vec{a}}{|\vec{a}|}=\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}.$

6. $\vec{a}=\alpha\vec{b}$ for some $\alpha\in R\Leftrightarrow\vec{a}\,//\,\vec{b}.$

Example 1：

Find the scalar and vector projection of *b* = <1,1,2> onto *a* = <-2,3,-1>.

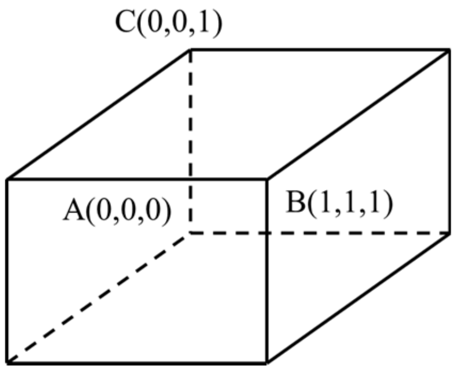
Solution：

$$\frac{a\cdot b}{|a|}=\frac{-2+3-2}{\sqrt{14}}=\frac{-1}{\sqrt{14}}$$

$$\frac{a\cdot b}{|a|}\frac{a}{|a|}=-\frac{1}{\sqrt{14}}\frac{<-2,3,-1>}{\sqrt{14}}=\left\langle\frac{1}{7},\frac{-3}{14},\frac{1}{14}\right\rangle.$$

Example 2：

Find the angle between a diagonal of a cube and one of its edges.



Solution：

$$\overrightarrow{AC}=<0,0,1>$$

$$\overrightarrow{AB}=<1,1,1>$$

$$\Rightarrow \cos\angle BAC=\frac{1}{\sqrt{3}}\Rightarrow \angle BAC=\cos^{-1}\frac{1}{\sqrt{3}}.$$