§11.5 Alternating Series

- **Who**： $\sum_{n=1}^{\infty}(-1)^{n+1}a_n = a_1 + (-a_2) + a_3 + (-a_4) + \ldots$

  $a_n > 0$ (or $a_n < 0$) for all n. (交錯級數，正負相間)

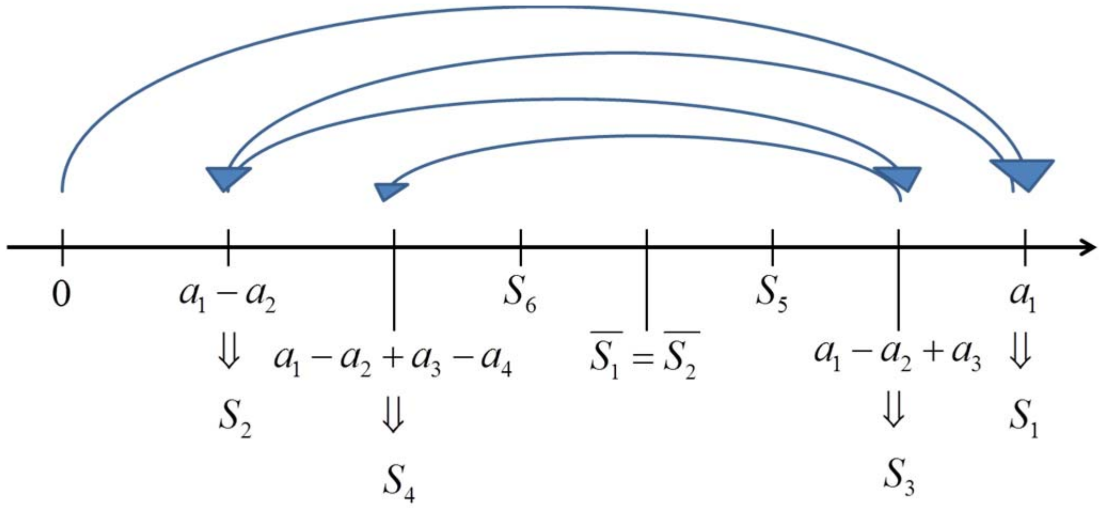
- **What**：

Theorem 1：

$$\left.\begin{array}{l}\text{(i) } a_n \text{ is decreasing.}\\ \text{(ii) } a_n \to 0 \text{ as } n \to \infty\end{array}\right\}(11.5\text{-}1)$$

$$\Rightarrow \sum_{n=1}^{\infty}(-1)^{n+1}a_n \text{ converges.}$$

- **Why**：



(i). $a_1 > 0$

(ii). $S_{2n} \to \overline{S_2}$ and $S_{2n+1} \to \overline{S_1}$

$$\Rightarrow S_{2n+1} - S_{2n} = a_{2n+1} \to 0$$
$$\Rightarrow \overline{S_1} = \overline{S_2}.$$

Example 1：

	Series	Conv./Div.
i.	$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$	Conv.
ii.	$\sum_{n=1}^{\infty}\frac{(-1)^n 3n}{4n-1}$	Div.
iii.	$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\ln n}{n}$	Conv.
iv.	$\sum_{n=1}^{\infty}(-1)^n\cos\frac{\pi}{n}$	Div.
v.	$\sum_{n=1}^{\infty}(-1)^n\sin\frac{\pi}{n}$	Conv.
vi.	$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(\ln n)^p}{n}$	Conv.

$$\left(f(x)=\frac{(\ln x)^p}{x} \Rightarrow f'(x)=\frac{(\ln x)^{p-1}(p-\ln x)}{x^2}<0,\ \text{for large } x. \right)$$

Theorem 2：(Error Estimate)

$\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ satisfies (11.5-1)

$|R_n|=|S-S_n|\leq a_{n+1}$

Proof：

$|R_n|=|S-S_n|\leq|S_{n+1}-S_n|\leq a_{n+1}.$