Hw9

1. (a) Give an example for each of the 4 types of generalized linear phase filters.

   (b) Let $h(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 2\delta(n-3) + \delta(n-4)$. Derive the Fourier transform of $h(n)$ and use its Fourier transform to show that it has generalized linear phase.

   (c) Let $H(z) = \sum_{n=0}^{M} h(n)z^{-n}$ be a filter with zeros at $z = -1, e^{j\pi/4}, j0.8, 0.9e^{j\pi 3/4}$. Find as many additional zeros as possible for $H(z)$ and mark the zeros of $H(z)$ on the z-plane when $h(n)$ has real coefficients and generalized linear phase.

2. Consider an LTI system $h(n)$ with input $x(n) = s(n)e^{j\omega_0 n}$, where $s(n)$ is a narrowband lowpass signal. We know the output $y(n)$ can be approximated as

$$y(n) \approx |H(e^{j\omega_0})|s(n - n_d)e^{j(\omega_0 n + \theta)},$$

   where $\theta = \angle H(e^{j\omega_0})$. Suppose $h(n)$ is real.

   (a) Find an approximate expression of $H(e^{j\omega})$ for $\omega \approx \omega_0$. Use the expression to find the output $y(n)$ when the input $x(n) = s(n)e^{-j\omega_0 n}$.

   (b) Find the output $y(n)$ when the input $x(n) = s(n)\cos(\omega_0 n)$.

3. (a) Draw the flow graph of 4-point FFT.

   (b) * Draw the flow graph of 6-point FFT.

4. * Consider an LTI system with impulse response $h(n)$. What can we say about $H(e^{j\omega})$ in (a) and (b).

   (a) $\sum_n h(n) = 0$.

   (b) $\sum_n h(n)(-1)^n = 0$.

   (c) Find out if some of the generalized linear phase filters satisfy one or both of the above conditions.