Hw6

1. Let $x(n) = \begin{cases} 1/4, & n = 0, 1, 2, 3 \\ 0, & \text{otherwise.} \end{cases}$

   (a) Find the DTFT of $x(n)$.

   (b) Find the N-point DFT of $x(n)$ for $N = 4$.

   (c) For $N = 4$, determine the periodic extension $\tilde{x}(n)$ and it's DFS coefficients.

   (d) Find $\tilde{X}(e^{j\omega})$, the Fourier transform of $\tilde{x}(n)$ in (c). How is $\tilde{X}(e^{j\omega})$ related to $X(e^{j\omega})$.

   (e) Repeat (b)-(d) for $N = 8$.

2. Suppose $x(n) = \begin{cases} 1/4, & n = -1, 1, \\ 0, & \text{otherwise.} \end{cases}$ Let $x(n)$ be a sequence with DTFT $X(e^{j\omega})$ and $X(k) = X(e^{j2\pi k/M})$, for $k = 0, 1, \cdots, M-1$, where M is some positive integer. Let $z(n)$ be the M-point IDFT of $X(k)$ for $n = 0, 1, \cdots M-1$.

   (a) Compute $X(k)$ for $M = 4$.

   (b) Compute $z(n)$, for $n = 0, 1, 2, 3$.

   (c) Find $z(n)$ for $n = 0, 1, 2, 3$ and $M = 4$ when $x(-1) = c_1, x(1) = c_2$ and $x(n) = 0$ otherwise.

   (d) Determine $z(n)$ in (c) for a general M.

3. * Let $x(n)$ be a discrete time sequence and $y(n)$ be given by $y(n) = x(n) * s(n)$, where $s(n) = \sum_{\ell=-\infty}^{\infty} \delta(n - \ell N)$ for some integer N. Determine the discrete time Fourier transform of $y(n)$ in terms of $X(e^{j\omega})$