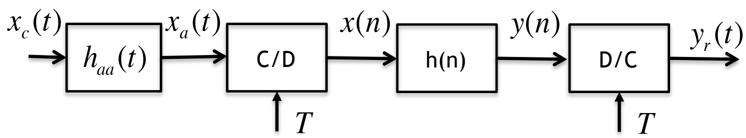
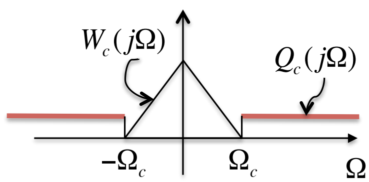
Hw5

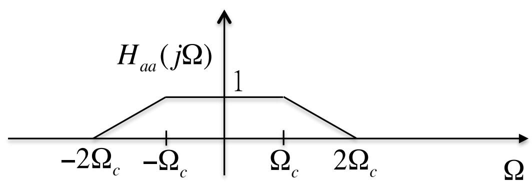
1. Consider the following system.



(a)



(b)



(c)

The input $x_c(t) = w_c(t) + q_c(t)$ is a sum of the desired signal $w_c(t)$ and noise $q_c(t)$, where $w_c(t)$ is band limited to $|\Omega| < \Omega_c$ as shown in (b) of the above figure. The anti-aliasing filter $H_{aa}(j\Omega)$ is a lowpass filter as shown in (c) of the above figure and $\Omega_s = 4\Omega_c$.

   (a) Plot $X_a(j\Omega)$ and $X(e^{j\omega})$.
   (b) Determine $H(e^{j\omega})$ so that $y_r(t) = w_c(t)$.
   (c) Is it possible to use a smaller sampling frequency for the given $H_{aa}(j\Omega)$ so that $y_r(t) = w_c(t)$? What is $H(e^{j\omega})$ in this case?
   (d) Suppose the sampling frequency is increased by a factor of 4. Design $H_{aa}(j\Omega)$ and $H(e^{j\omega})$ so that $y_r(t) = w_c(t)$? Are there different solutions?

2. Let x be a random variable satisfying $0 < x < 2$. Design a 2-bit uniform quantizer Q for x. Let $\hat{x}$ be the quantizer output $\hat{x} = Q(x)$.

   (a) Plot $Q(y)$ for $0 < y < 2$.
   (b) Find the quantization error $e = \hat{x} - x$ when $x = 0.2$ and $x = 1.4$.
   (c) Determine the variance of e when x is uniformly distributed over the interval $(0, 2)$.

3. * Consider the system in problem 1. Suppose the sampling frequency is increased by a factor of 4. Replace the discrete time system $h(n)$ by a sampling rate reduction system (i.e., a lowpass filter followed by a decimator). Design $H_{aa}(j\Omega)$ and change the sampling period of the D/C so that $y_r(t) = w_c(t)$ and $H_{aa}(\Omega)$ has the largest possible transition band.