Hw 10

1. Let
$$H(z) = \frac{1-(1.2)^2 z^{-2}}{(1-0.5e^{j\pi/4}z^{-1})(1-0.5e^{-j\pi/4}z^{-1})}.$$

   (a) Find a minimum phase filter $F(z)$ that has the same magnitude response as $H(z)$

   (b) Find $G(z)$ so that when an input is applied, the output of $G(z)H(z)$ has the same magnitude response as the input.

   (c) Express $H(z)$ as a product $H(z) = A(z)B(z)$, where $A(z)$ has generalized linear phase and $B(z)$ has minimum phase.

2. Let $H(z) = \prod_{k=1}^{N} \frac{z^{-1}-d_k^*}{1-d_k z^{-1}}$ be an N-th order allpass filter. Express $H(z)$ as $\frac{A(z)}{B(z)}$ and let $B(z) = 1 + \sum_{k=1}^{N} b_k z^{-k}$.

   (a) How are the coefficients of $A(z) = \sum_{k=0}^{N} a_k z^{-k}$ related to those of $B(z)$?

   (b) How does the answer change when $h(n)$ is real?

   (c) Suppose $h(n)$ is real? and the coefficients of $A(z)$ and $B(z)$ satisfy the property in (b). Is $H(z)$ allpass?

3. Suppose $H(z)$ is a causal and stable allpass filter with $|H(e^{j\omega})| = 1$. Let $x(n)$ be a causal input and $y(n)$ be the output.

   (a) Show that $\sum_n |h(n)|^2 = 1$.

   (b) Show that $\sum_n |x(n)|^2 = \sum_n |y(n)|^2$.

   (c) * Show that $\sum_{n=0}^{L} |x(n)|^2 \geq \sum_{n=0}^{L} |y(n)|^2$ for all L.

4. **MATLAB bonus problem.** (Due 12/31) In this assignment, you are asked to design a filter to filter out noise. At the class webpage, you can find 'jammed.mat'. After loading the data using `load`, you will find a vector `y` and a scalar F_s. The vector `y` contains a contaminated music signal that is sampled with sampling frequency F_s. Design a filter $h(n)$ to filter out the jamming noise but retain as much signal as possible. (Related Matlab commands: `load, kaiserord, fir1, conv, fft, soundsc, semilogy`.)

   (a) Comment on how the filter is designed.

   (b) Plot the magnitude response of the filter you have designed.

   (c) Extract a few hundred samples from the first note of the output signal and plot it.

   Useful iterative steps for the design of the filter $h(n)$.

   (i) Examine the contaminated signal `y` in the time and frequency domains to make a guess of the desired filter specifications and design the filter accordingly.

(ii) Apply filtering and examine the output in both time and frequency domains and also listen to it.

(iii) If it is not satisfactory, adjust the specifications, redesign the filter and go back to step (ii). Stop if the output is satisfactory.